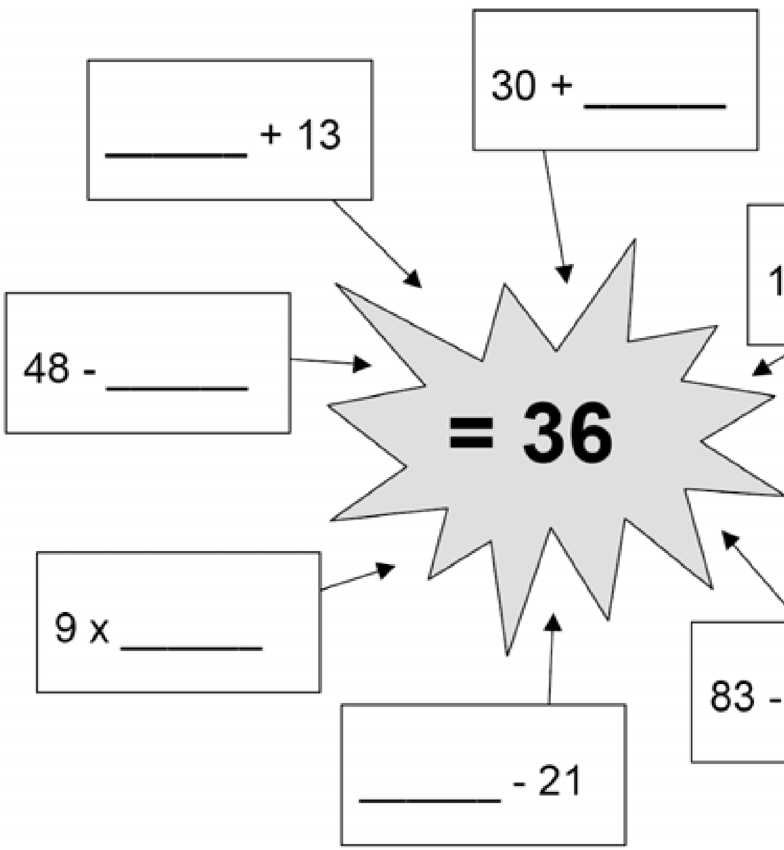
The Answer is 36

This problem gives you the chance to:
- work with number calculations to get the answer 36

1. Fill in the missing numbers so that the answer is **always 36**.



2. Fill in the gaps below to make the answer 36.
   You may use these signs: + -

47 ____ 35 ____ 24 = 36

9

The Answer is 36		Rubric
The core elements of performance required by this task are: • work with number calculations to get the answer 36 • Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers, start top right in a clockwise direction: **6** **19** **47** **57** **4** **12** **23**	7 x 1	7
2. Gives correct answers: **-** and +	2	2
Total Points		**9**

3rd Grade – Task 3: The Answer is 36

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess?

____

____

____

Look at the three addition problems. How do you think the order that the blank appeared (first or second) effected the strategies that students used to solve the problem? Do you think one blank position might be more difficult than the other? Why?

Look at student work for the 3 addition problems. For 17 + _____, how many of your students put:

19	20	29	36	14	Other

What might have caused their confusion?

Now look at _____+ 13, how many of your students put:

23	20	53	17	24	Other

Now think about the 3 subtraction problems. What strategies might students use to solve these problems? Do you think the strategies would be different for solving the problems with the blanks first or second? What other factors might make some of these more difficult than others?

Look at student work for the 3 subtraction problems. For 83 - _____, how many of your students put:

47	59	43	20	57	119	Other

What misunderstanding might have led to some of the common errors?

Look at _____- 21. How many of your students put:

57	15	52	36	47	Other

Are the misunderstandings different from those in the previous problems?

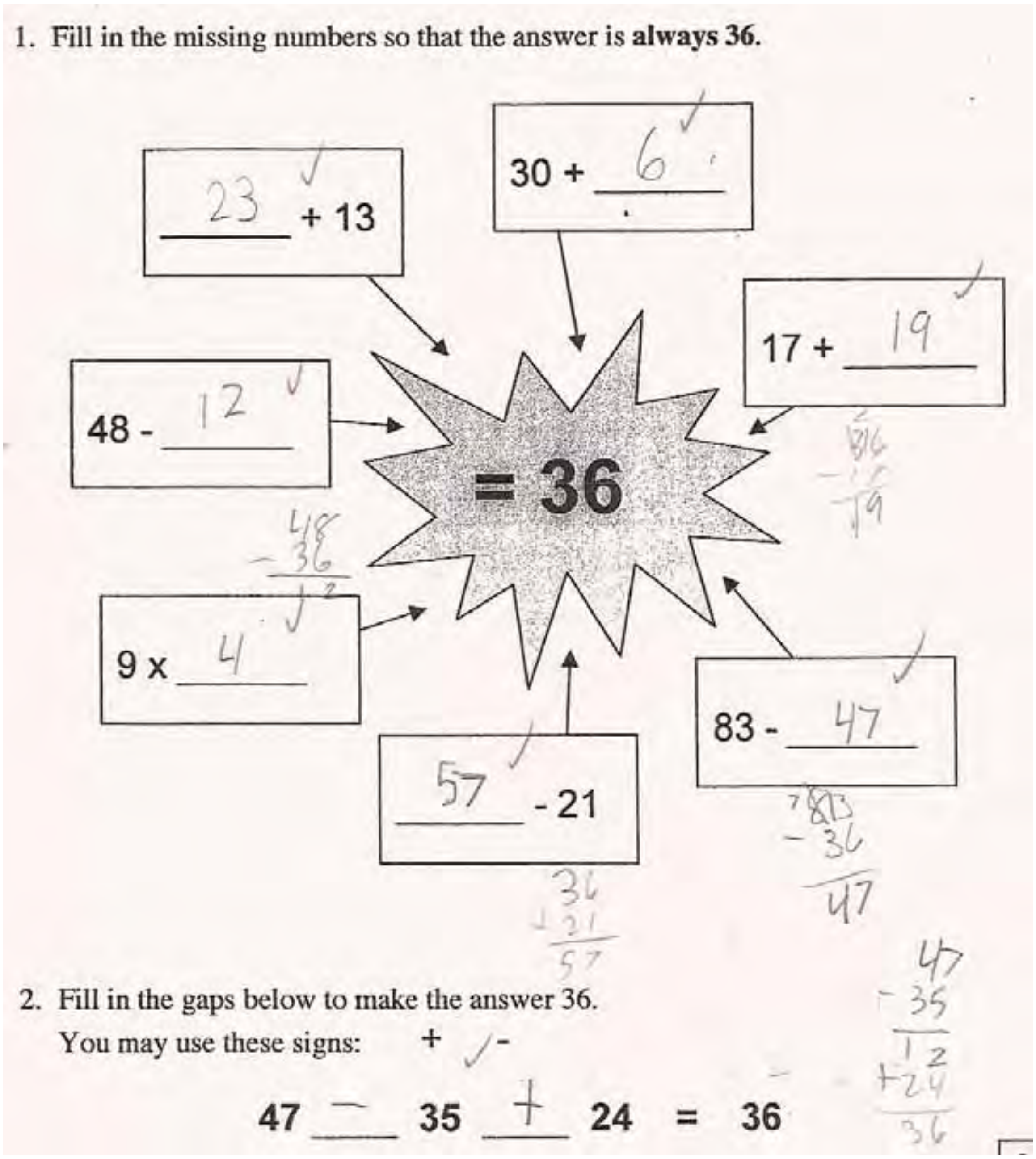
While the students were not required to show their work for this task, did your students show their strategies on the side?

- What evidence do you see in their work about misunderstanding place value?
- Misunderstanding order of operations?
- Misunderstanding equality? What is your evidence?

Looking at Student Work

This task looks at student understanding of the relationship between addition and subtraction, place value, order of operations, and making sense of equality. While the task does not require students to show work, many students use the space to show their thinking or make calculations. Student A shows that inverse operations can be used to solve missing part problems for addition and subtraction.

Student A



1. Fill in the missing numbers so that the answer is **always 36.**

23 + 13

30 + 6

17 + 19

48 - 12

= 36

9 x 4

57 - 21

83 - 47

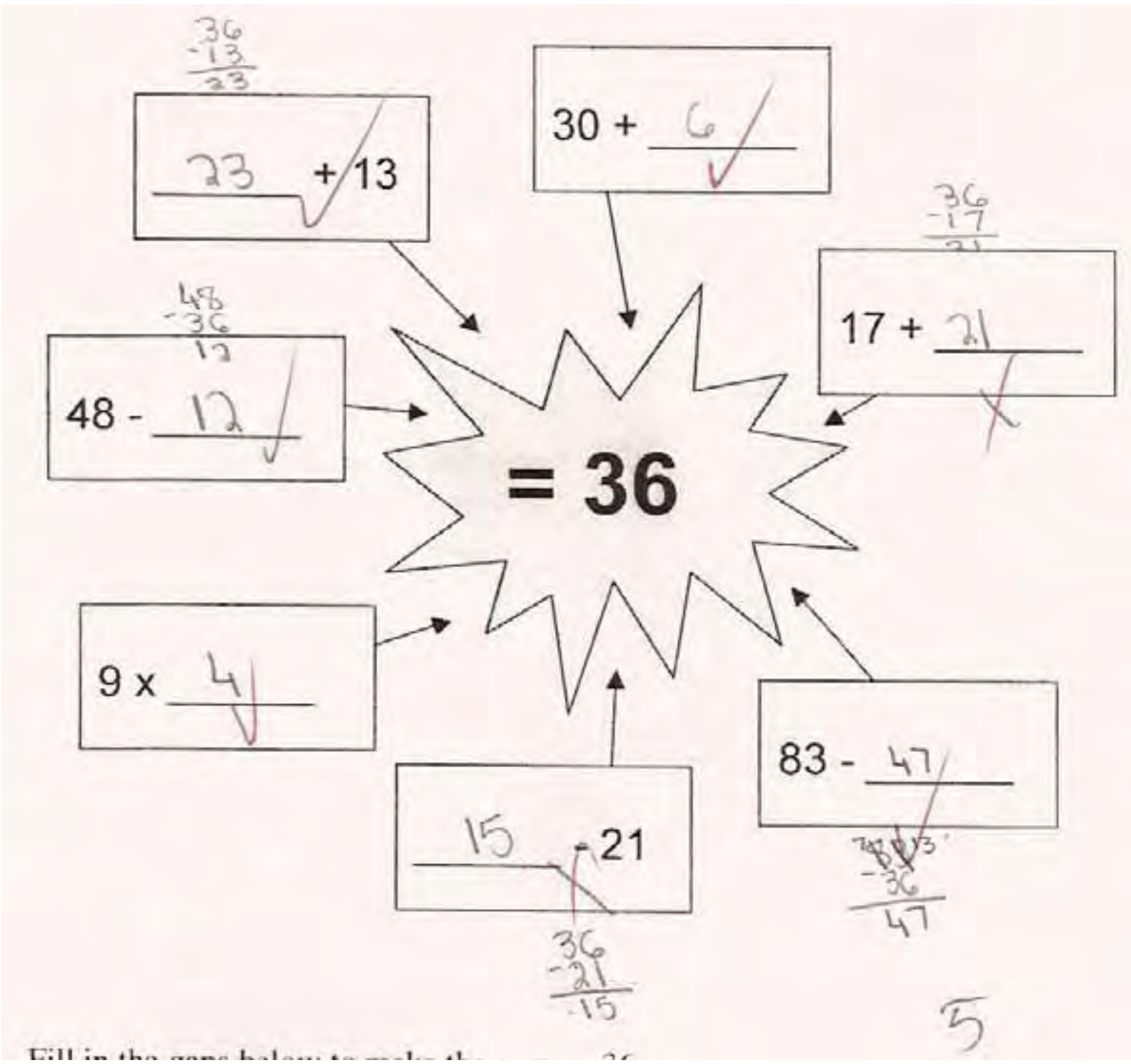
2. Fill in the gaps below to make the answer 36.

You may use these signs: + -

47 − 35 + 24 = 36

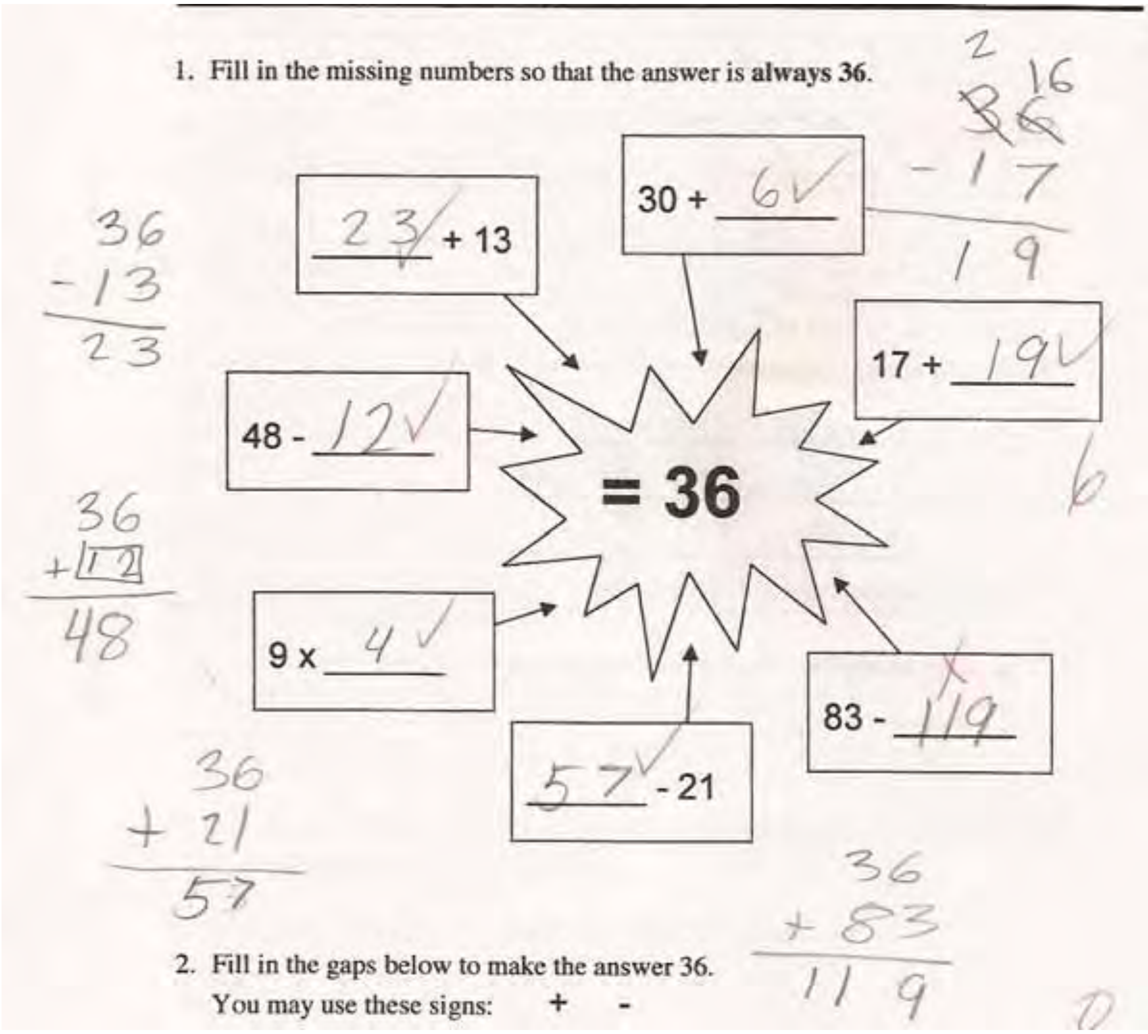
Student B tries to solve all the problems using subtraction. While this works for 86-___ *or* 48- ____, it is not useful for solving a problem in the format ____minus a quantity. The student doesn't seem to make sense of the idea that to get an answer of 36, the initial quantity, what you start with before the subtraction, must be larger than 36. The student is not thinking about how the process of subtraction effects the change in the initial quantity. *What activities might help the student develop better number sense?*

Student B



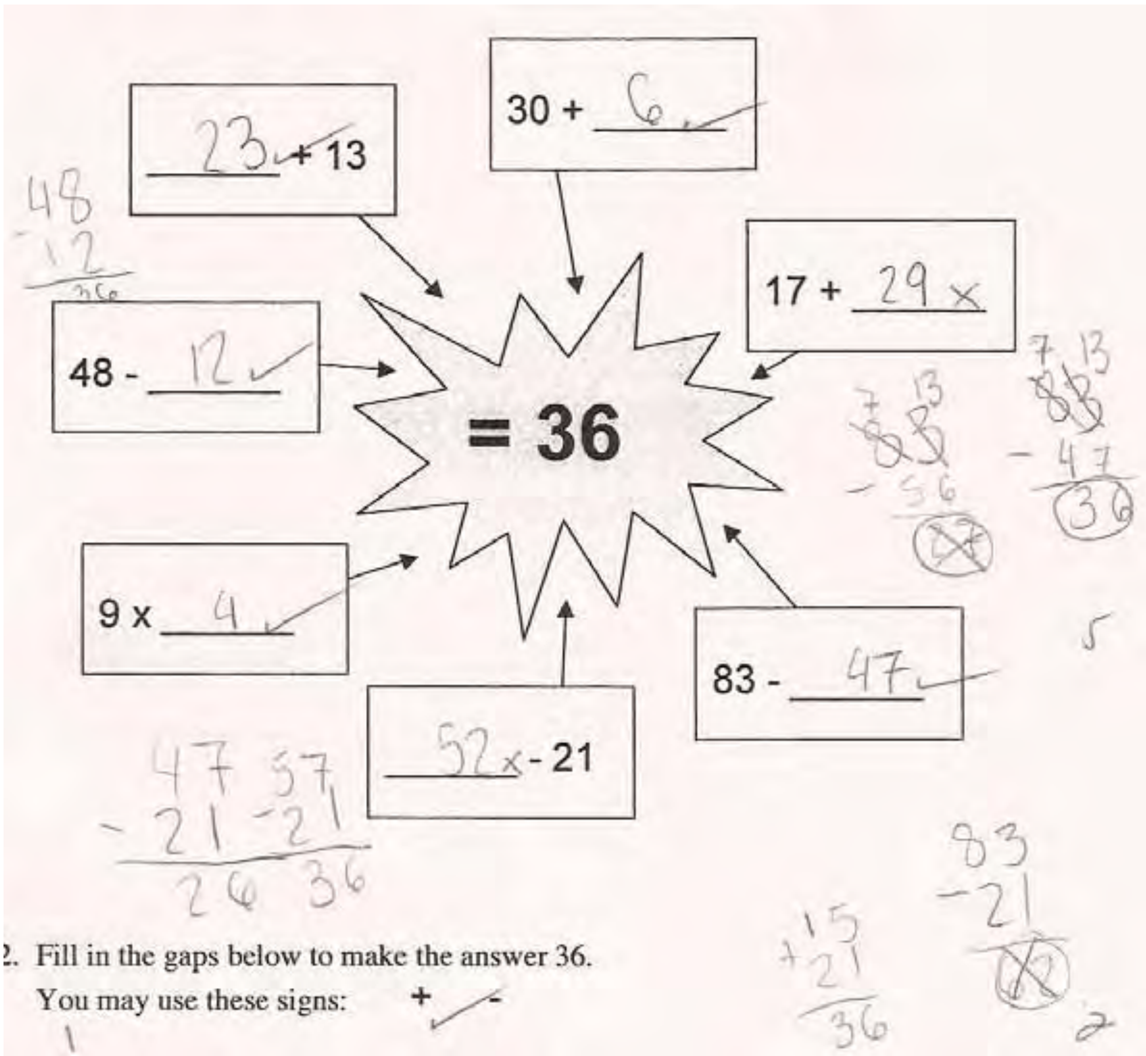
Student C has a similar problem reasoning about the effects of operation on the solution. In thinking about ____-21, the student successfully uses addition to find the missing number. However this process does not work for 83- _______. *Why doesn't the inverse operation apply to this problem as well? What would you want the student to be thinking about as she worked this task?*

Student C



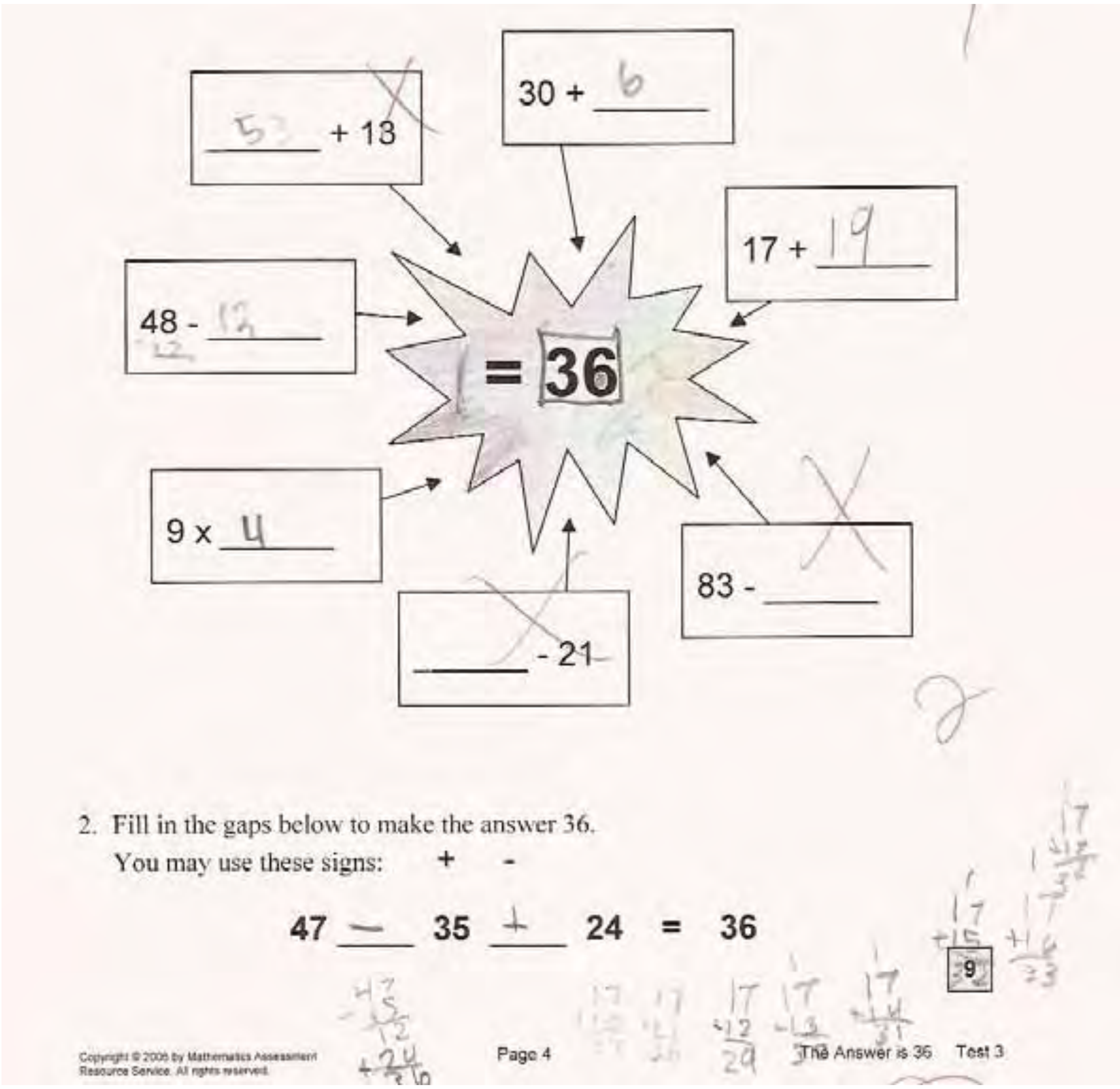
Student D uses guess and check to try and find the solution to many parts of the task. *What does the student need to understand about the operation of addition and subtraction to be able to use a more reliable method? Do you think this student would benefit from number talks? How might number talks contribute to the student's understanding of place value?*

Student D



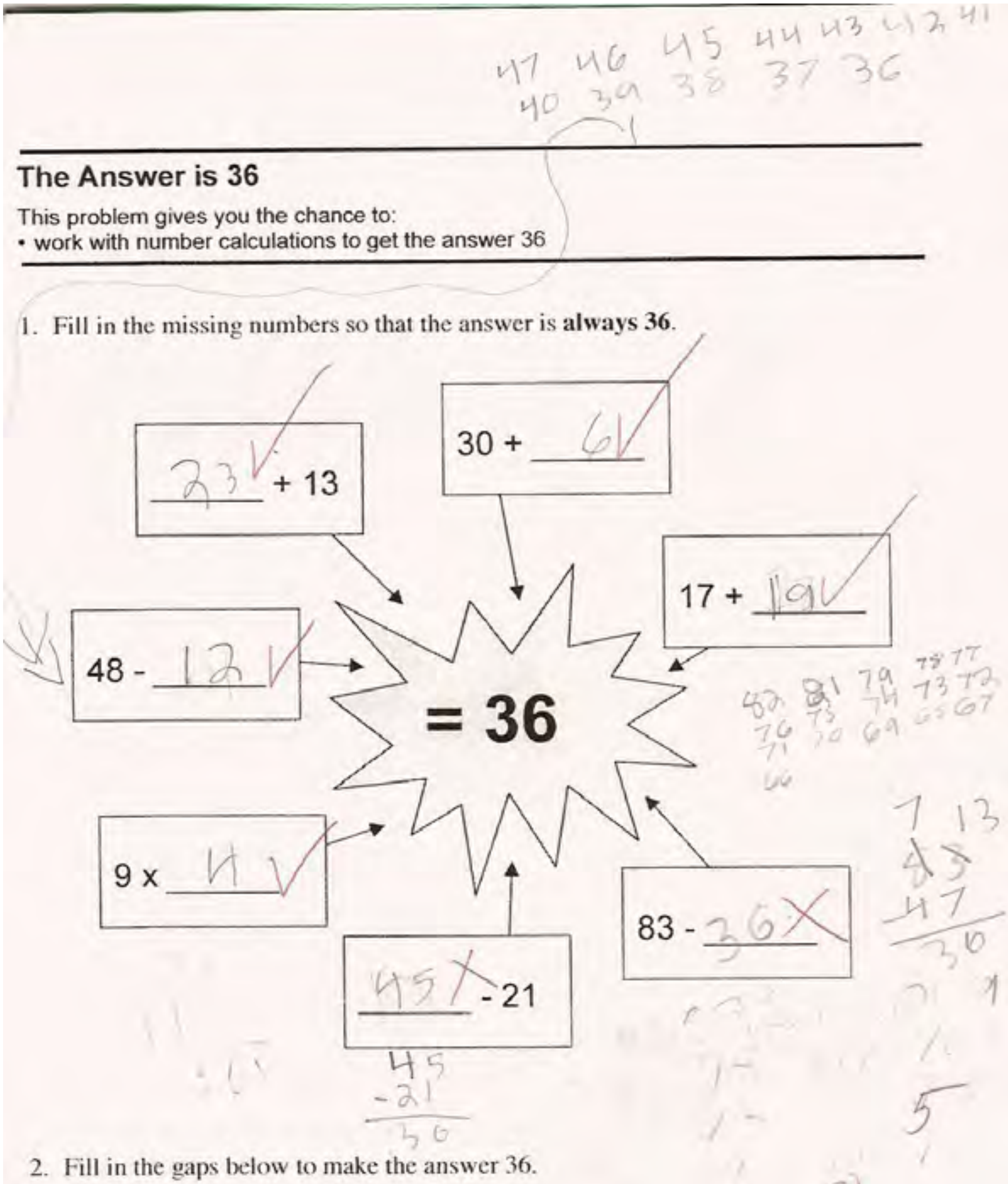
Student E is also using a guess and check strategy to find 17 + ____ =36. Notice that this student cannot reason about the distance between the original guess and the final answer. The student's guesses go up only one number at a time. *What kinds of experiences or questions might help this student think more about the global picture?*

Student E



Some students have not moved to computation with subtraction. Student F is still relying on a counting up or counting down strategy to find the difference. While this is useful to learn about the process at lower grade level, third graders should be able to usemore efficient strategies. Number talks might help this student move from going backwards one step at a time to moving in groups of 10. Some sort of intervention is needed to help this student move beyond this strategy.

Student F



Third Grade

3rd Grade Task 3 The Answer is 36

Student Task	Work with number calculations to get the answer 36 using addition, subtraction and multiplication.
Core Idea 2 Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.** • Understand multiplication as repeated addition, an area model, an array, and an operation on scale. • Develop fluency in adding and subtracting whole numbers.

The mathematics in the task:
- Ability to add double digit numbers, with or without carrying.
- Ability to subtract double digit numbers, with or without regrouping.
- Ability to multiply single digits.
- Ability to use inverse operations to solve problems with addition and subtraction and show understanding of how operations effect the size of the answer.

Based on teacher observation, this is what their graders knew and were able to do:
- Solve for missing addends
- Add with accuracy
- Multiply by one digit numbers
- Reason about order of operations and fill in missing operation signs

Areas of difficulty for third graders:
- Order of operations
- Place value and regrouping
- Finding missing information in subtraction problems

Strategies used by successful students:
- Inverse operations
- Calculations
- Guess and check
- Count backwards or count forwards
- Show their work
- Checking their answers, e.g. 36 + 21 = 57 /57-21=36

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 3

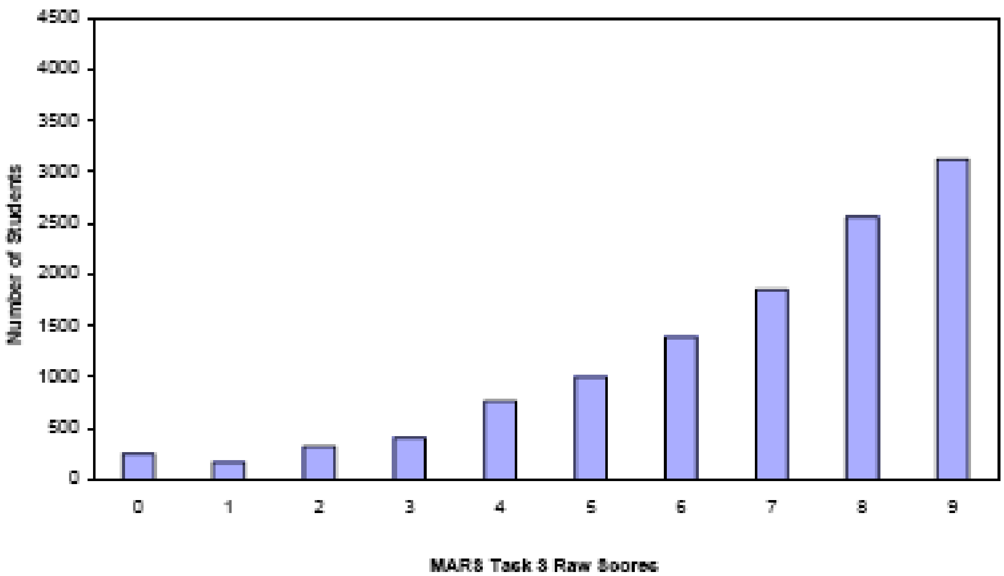
Task 3 - The Answer is 36

Mean: 6.75 StdDev: 2.25

Table 17: Frequency Distribution of MARS Test Task 3, Grade 3

Task 3 Scores	Student Count	% at or below	% at or above
0	261	2.2%	100.0%
1	175	3.7%	97.8%
2	319	6.4%	96.3%
3	410	9.8%	93.6%
4	757	16.2%	90.2%
5	998	24.6%	83.8%
6	1393	36.4%	75.4%
7	1857	52.1%	63.6%
8	2560	73.7%	47.9%
9	3121	100.0%	26.3%

Figure 26: Bar Graph of MARS Test Task 3 Raw Scores, Grade 3



The maximum score available on this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, about 94%, could find the missing addends for 30 + ____ and for ___+ 23 and knew the fact that 4 x 9 =36. About 90% could also correctly fill in the missing operation signs in the bottom part of the task. More than half the students, 63%, could find all the missing addends; they may have struggled with one or more of the subtraction problems. 83-____ seemed to be the most difficult. 29% of the students understood the operations of addition and subtraction and could use inverse operations to find missing parts. Only 2% of the students scored no points on this task. Half of the students in the sample with this score attempted the task.

The Answer is 36

Points	Understandings	Misunderstandings
0	Only 2% of the students scored zero. Half of them attempted the task.	
4	Students could find the missing addends for 30 + _____ and for ___+ 23 and knew the fact that 4 x 9 =36.	Students had difficulty with 17+___=36. 4% made a calculation error and wrote 20. 3% had difficulty with place value and put 29 instead of 19.
6	Students could find all the missing addends and knew 9x 4.	Students had difficulty thinking about how to find the missing parts in subtraction problems. For ___-21, 10% of the students subtracted 21 from 36 to get 15 as the missing number. 2% thought the answer was 36 and 2% thought it was 47.
8		Students still struggled with subtraction. The most difficult part was 83-____=36. 9% put 59. 4% put 43. 2% put 119.
9	students understood the operations of addition and subtraction and could use inverse operations to find missing parts.	

Implications for Instruction

Students need help with subtraction. They should know a variety of strategies for thinking about addition and subtraction of 2-digit numbers. An understanding of part/part/whole relationships in addition and subtraction allows students to be more flexible in finding missing parts in addition and subtraction and to see the connection between the two. Work with the number line or empty number line help students make sense of size and relationship of the numbers. Thinking about the positions on the number lines might eliminate some of the mistakes involved in the traditional algorithms of carrying and regrouping. The bar model also helps develop these relationships and gives students a visual picture of the action of the operation. Learning to understand the operations is as important as the computational facts. Use of number talks as class warm-ups can familiarize students with how these models work.

Liping Ma, in her book Knowing and Teaching Elementary Mathematics, talks about the importance of emphasizing composing and decomposing numbers using addition and subtraction on numbers between 0-20 or going around 10. Some students would benefit from some deep work on this area. Developing a clear understanding of how the operations work with these numbers, makes moving to larger numbers almost self-teaching. Work with the dot cards from Kathy Richardson, number talks, even games like dominos can help students start to visualize how the base-10 system works and make sense of addition and subtraction as they develop their own strategies for adding and subtraction based on the composition and quantity of the numbers.

Ideas for Action Research

Cognitive Dissonance

Many researchers think misconceptions are best dealt with by creating situations where students need to confront their misunderstandings in a context that helps them discover why those ideas are incorrect. Consider giving your students a puzzle like:

> *Sally says that to find the missing part in a subtraction problem, you always use the opposite operation addition. She proves her point by showing that for:*
> ***____- 21= 36,*** *so* ***21 +36*** *equals the missing part is* ***57.***
>
> *Sam says that to find the missing part in a subtraction problem, you always use subtraction. His example is:*
> ***83-___=36,*** *so* ***83-36*** *the missing part 47.*
>
> *Who is right? Why? How can you convince someone?*

Have students work in pairs to find a convincing argument. If students don't come up with the idea of using a model on their own, ask them if they could use a number line or bar model to help make their justification. Now give students some similar missing parts problems to solve. Do they have better strategies? Are they more accurate?

Going Around Ten

Read Liping Ma's Chapter about Subtraction with Regrouping: Approaches to Teaching the Topic. See if you can use the ideas sparked by the Chinese teachers to plan some number talks. Work with students in small groups for about 15 minutes a day for a week or two. Maybe try some groups with just lower achieving students and some groups with a mix. How do students think about composing or decomposing the numbers? Does this work help students develop a better understanding of arithmetic? What changes do you notice as students work with these ideas? Was there a difference in how students thought in the different groups? How did the lower achieving students in both groups progress in their understanding? Give an anecdote that shows why this was helpful.

Use of models

Pick a model, like the empty number line, and give students some opportunities to solve addition and subtraction problems using the number line. Have students explain their strategies? Notice which students are moving one step at a time and which students can move or think in larger groups, like fives, tens or larger. As you work with this idea for a couple of weeks, use your seating chart to keep track of strategies used by different students. Are more students thinking in groups or "chunks" over time? As they get better, see if this model can be expanded to thinking about multiplication or division.

Performance Assessment Task
The Answer is 36 Grade 3
The task challenges a student to use knowledge of operations and their inverses to complete number sentences that equal a given quantity. A student must demonstrate understanding of concepts involved in the meaning of operations. A student must develop fluency in adding and subtracting whole numbers as well as understand multiplication as repeated addition. The student must develop an understanding of the inverse relationship between addition and subtraction as well as between multiplication and division and be able to apply this understanding to problems. A student must use understanding of the size of the given quantities to determine the appropriate operation or the size of the missing quantity.
Common Core State Standards Math - Content Standards
<u>**Operations and Algebraic Thinking**</u> **Represent and solve problems involving multiplication and division.** 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 + ? **Understand properties of multiplication and the relationship between multiplication and division.** 3.OA.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
3	2006	9	4	91%