Squares and Circles

This problem gives you the chance to:
- work with perimeter and circumference of squares and circles
- use and interpret line graphs and their equations

The points on this graph show the perimeters of squares of different sizes.

For example, a square with sides 2 inches long has a perimeter of 8 inches.

1. What is the perimeter of a square with sides 5 inches long?

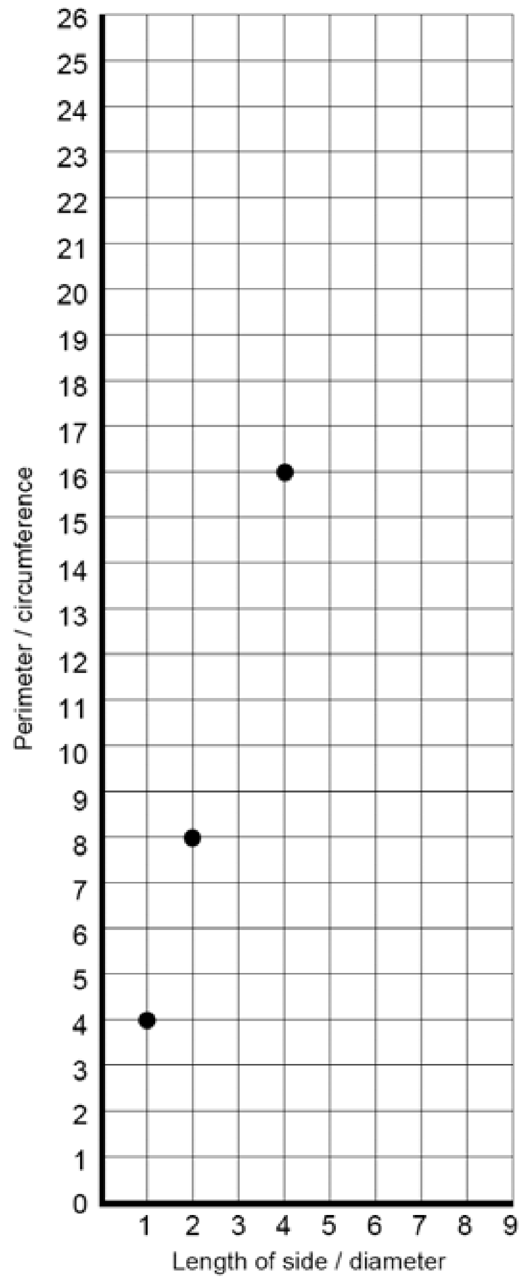
   ____

   Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?

   ____

   Mark a point for this square on the graph.



3. Draw a line through the points on the graph.

   (a) Explain why the line passes through the point (0, 0).

   ____

   ____

   (b) Explain why the line is straight.

   ____

   ____

4. Draw a circle around the correct equation of the straight line on the graph.

$$x + y = 4 \qquad y = x + 4 \qquad y = 4x \qquad y = \frac{x}{4}$$

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
   Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

   Same:

   ____

   Different:

   ____

9

Squares and Circles		Rubric
The core elements of performance required by this task are: • work with perimeter and circumference of squares and circles • use and interpret line graphs and their equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **20** (inches) and correct point marked on graph.	1	1
2. Gives correct answer: **3** (inches) and correct point marked on graph.	1	1
3. Correct line drawn.	1	
(a) Gives correct explanation such as: The perimeter is zero if the side length is zero.	1	
(b) Gives correct explanation such as: The perimeter is always four times side length. **or** The perimeter is proportional to side length.	1	3
4. Gives correct answer: $\boldsymbol{y = 4x}$	1	1
5. Correct points marked and line drawn.	1	1
6. Writes correct statements such as:		
Both lines go through (0,0)	1	
The line for the squares is steeper than the line for the circles.	1	2
Total Points		**9**

Squares and Circles

Work the task. Look at the rubric. What do you think are the major difficulties students might have with this task?

In part one and two, a few students had trouble with graphing. They may have made zigzagging lines because one of their points was off or connect the points to one of the axes. Did any of your students have difficulty with the graphing issue?

Almost 35% of the students didn't graph the line for diameter/circumference. If you get a chance, interview a few of your students. Did they give up before they got that far in the task? Did they overlook the instructions in part five? Do you think trying to graph numbers with decimals confused them?

For part 3a, many students thought that lines or all lines go through zero or that it goes through zero because it slopes. They did not relate the origin to the context. What were the common misconceptions that you found in your students' work?

For part 3b, many students thought that the points were linear because they lined up that way or because there was a pattern? What were the common misconceptions that you found in your students' work?

What kinds of graphing activities have your students done this year? What opportunities have they had to graph from a context and discuss how the context effects the shape of the graph?

Look at student work in part six. How many of your students wrote answers that discussed:

- Slope (going upward, quantifying amount, steepness of the line)
- Origin
- Whole numbers/decimals
- Nonmathematical ideas: easy to read, ones easy ones hard, the dots are large

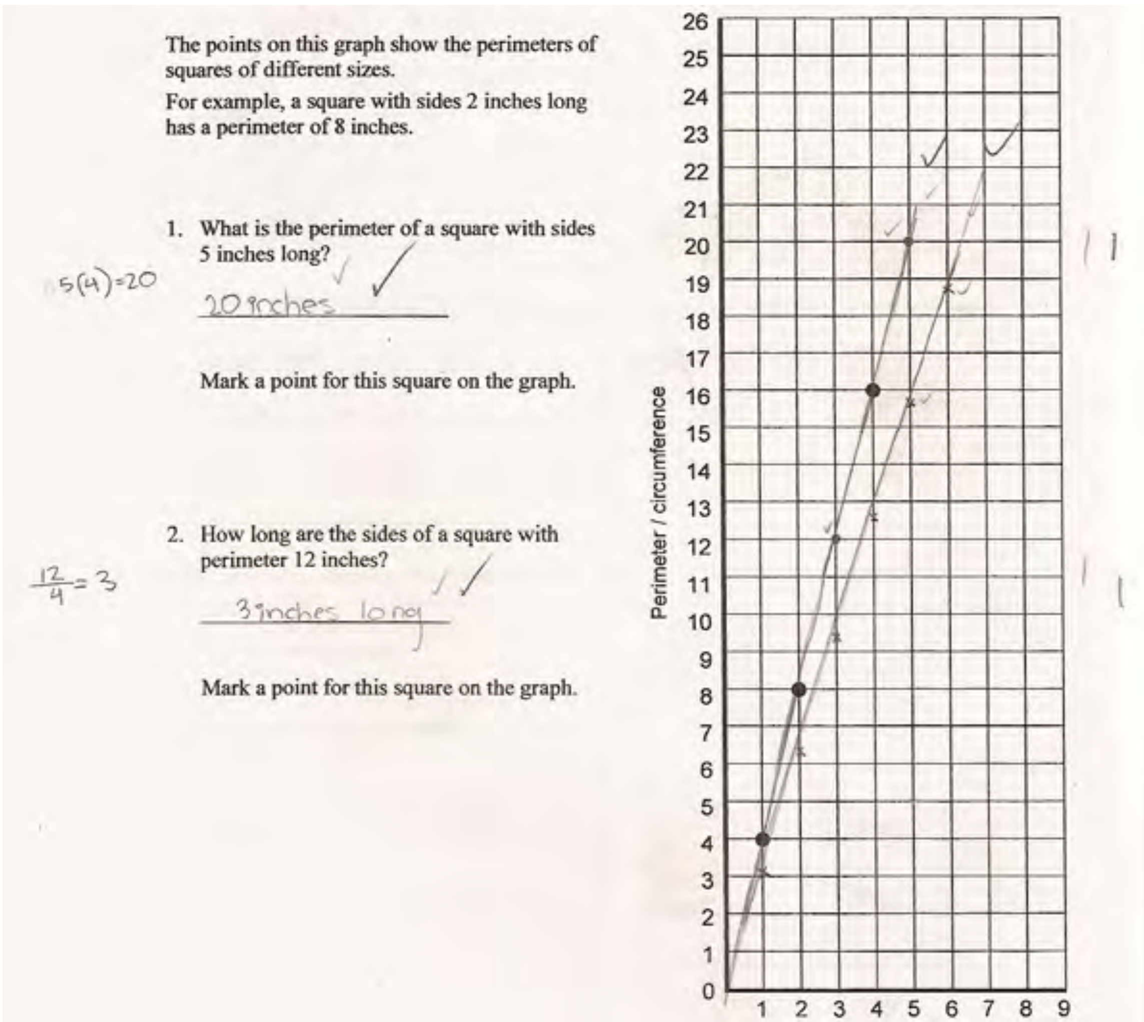
Did your students know what where critical or important things to consider when comparing and contrasting graphs?

What are some of the implications for instruction?

Looking at Student Work on Squares and Circles

Student A shows how he calculated perimeter and side length. The student is able to relate the origin to the context of the problem in 3a. Notice the reference to origin and slope in part 6.

Student A



The points on this graph show the perimeters of squares of different sizes.

For example, a square with sides 2 inches long has a perimeter of 8 inches.

1. What is the perimeter of a square with sides 5 inches long?

5(4)=20

20 inches

Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?

$\frac{12}{4}=3$

3 inches long

Mark a point for this square on the graph.

Student A, continued

(a) Explain why the line passes through the point (0, 0).

because if a square has sides 0 inches long than the perimiter is 0 inches

1 1

(b) Explain why the line is straight.

because squares sides are alway equal lengths and the longer the sides the larger the perimeter

1 1

4. Draw a circle around the correct equation of the straight line on the graph.

$x+y=4$ $\quad y=x+4$ $\quad y=4x$ $\quad y=\frac{x}{4}$

1 1

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
Join the Xs with a straight line.

1 1

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

The lines both pass through (0,0)

1 1

Different:

The squares perimeter gets larger quicker than a circles circumference

Student B has an almost perfect paper, but then when finding differences looks at the numbers plotted rather than properties of the line or graph.

Student B

Same:

in a staight line and the origin is 0 length and perimeter

1

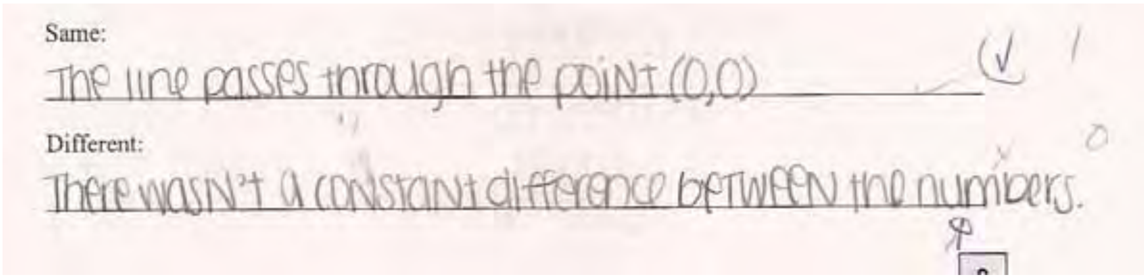
Different:

squares are on whole numbers, X's are on decimal spots

0

Student C also has a score of 8 points. The student is not thinking about the issue of rounding in finding circumference, but instead notices that for the values in the table the difference is slightly different between points. *What kind of activity would help students understand level of accuracy as it relates to this type of function?*

Student C


Same:

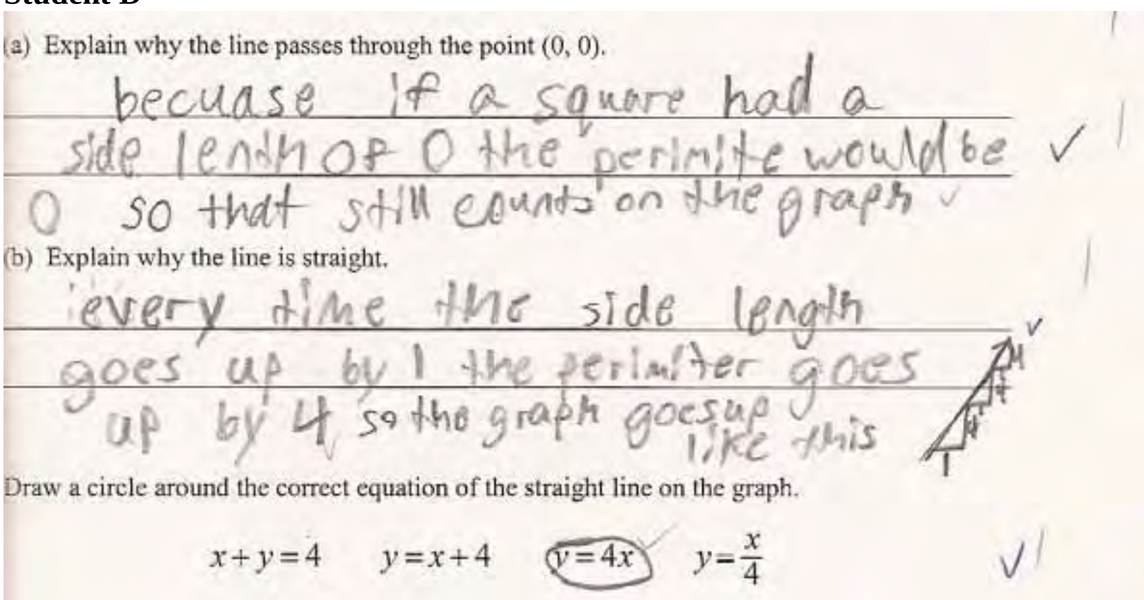
The line passes through the point (0,0)

Different:

There wasn't a constant difference between the numbers.

Student D has a score of 9. Notice the drawing to explain why the line is straight.

Student D


(a) Explain why the line passes through the point (0, 0).

becuase if a square had a side lenth of 0 the perimite would be 0 so that still counts on the graph

(b) Explain why the line is straight.

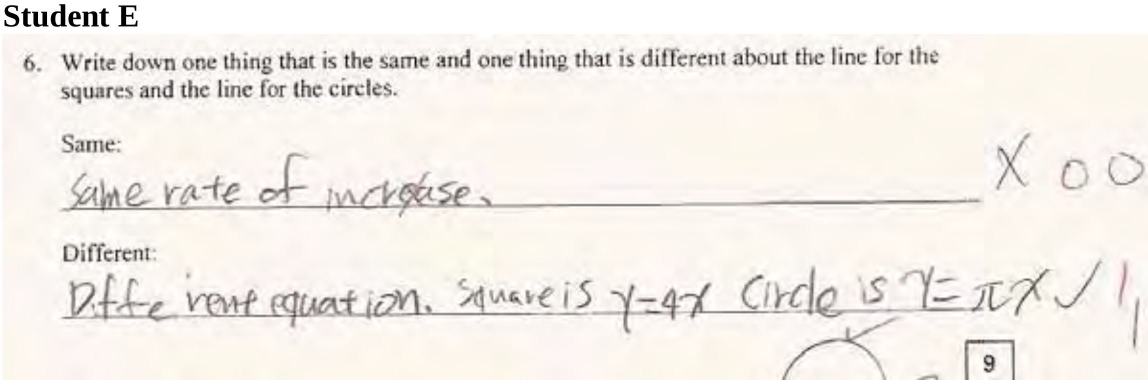
every time the side length goes up by 1 the perimter goes up by 4 so the graph goes up like this

Draw a circle around the correct equation of the straight line on the graph.

$x+y=4$ $y=x+4$ $y=4x$ $y=\frac{x}{4}$

Student E is another example of an "8". The student understands the properties that should be compared and is able to give the equations. However the student does not see that the rate of increase is different for circles than squares.

Student E


6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

Same rate of increase.

Different:

Diffe rent equation. Square is $y=4x$ Circle is $y=\pi x$

As scores drop from 8 or 9, there is a change in the types of arguments presented in part three and six. Student F is able to calculate the perimeter and side length in part one and two and draw the graphs for circles and squares. While the student can graph, the understanding of meaning and purpose seem to get lost. The student's line for circle misses the origin by a hair. *How would you compare the reasoning in this paper to that of the previous students?*

Student F

(a) Explain why the line passes through the point (0, 0).

Because all of the perimeters start somewhere. X X

(b) Explain why the line is straight.

Because the data follows a pattern. X X

4. Draw a circle around the correct equation of the straight line on the graph.

$x + y = 4$ $y = x + 4$ $y = 4x$ $y = \frac{x}{4}$

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

They follow a pattern and make a straight line.

Different:

The points drawn into a line to not pass through (0, 0). X

9

While Student G is able to calculate the perimeter and side length in part one and two, the student does not plot the points for either correctly. Notice that there is an idea that straight lines start at (0,0), which is prevalent in many papers. *Why do you think students have this idea? What kinds of experiences would help them to understand why not all lines go through the origin? What do students need to understand to know why a function will be linear or nonlinear?*

Student G

1. What is the perimeter of a square with sides 5 inches long?

5 $5 \times 4 = 20$

P= 20 in.

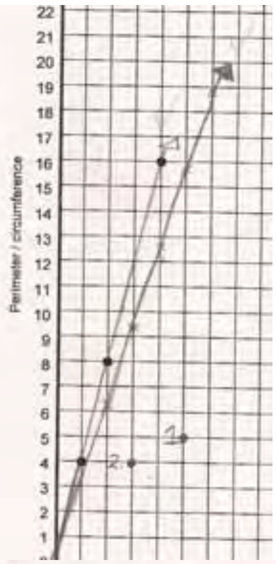
Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?

$4\overline{)12}$ 3

3 in. long

Mark a point for this square on the graph.



(a) Explain why the line passes through the point (0, 0).

The line pass through point (0,0) because it is a straight line

0

(b) Explain why the line is straight.

The line is straight because it is a square and all sides must be equal, therefore straight

0

4. Draw a circle around the correct equation of the straight line on the graph.

$x + y = 4$ $y = x + 4$ $y = 4x$ $y = \frac{x}{4}$

1

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

1

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

They both go through point (0,0) and they are both diagonal lines

1

Different:

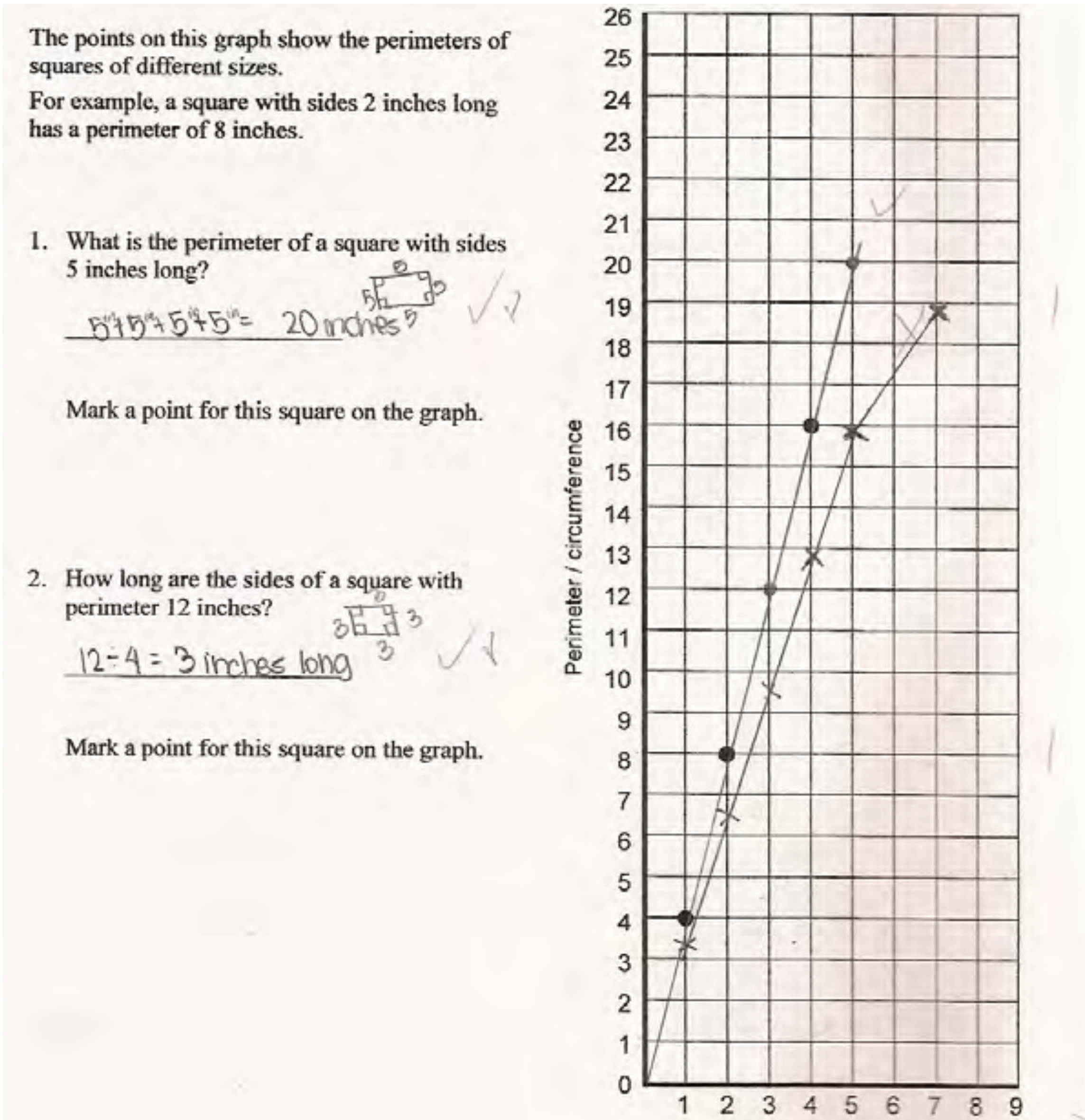
The diameter/circumfrence one has different plot numbers and it is on a different pattern like the circumfrence one goes up 3.2 up 3.1 and so on, but the perimeter/sides one goes up by 4 every time.

1

$\boxed{9}$ 6 7

Student H is able to find perimeter and side length, but does not notice that the final pointfor circles is graphed incorrectly. The student, like many others with scores of 4 or less, is no longer focused on the mathematics. Student H is concerned primarily with accuracy. The student understands that the square pattern is increasing by 4 every time and gives the recursive equation rather than the seeing the multiplicative relationship.

Student H



The points on this graph show the perimeters of squares of different sizes.

For example, a square with sides 2 inches long has a perimeter of 8 inches.

1. What is the perimeter of a square with sides 5 inches long?

   5"+5"+5"+5"= 20 inches

   Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?

   12÷4 = 3 inches long

   Mark a point for this square on the graph.

Student H, continued

(a) Explain why the line passes through the point (0, 0).

Because it has to start there and you need it to make your line accurate

(b) Explain why the line is straight.

Because the measuring are right, very acurate. And because you add 4 and x to get y

4. Draw a circle around the correct equation of the straight line on the graph.

$$x + y = 4 \qquad y = x + 4 \qquad y = 4x \qquad y = \frac{x}{4}$$

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

1-5 has a straight line like the square

Different:

the circle graph has a not so straight graph it goes like this while the square goes like this.

9

As students get to scores of 3 or less, many students do not attempt page two of the task. They seem unwilling to even attempt making explanations. For those students who attempted this part, work might look like Students I and J. *How would you compare their responses to 8's and 9's? to the 4,5,6's? What is the difference in their mathematical thinking?*

Student I

The points on this graph show the perimeters of squares of different sizes.
For example, a square with sides 2 inches long has a perimeter of 8 inches.

1. What is the perimeter of a square with sides 5 inches long?

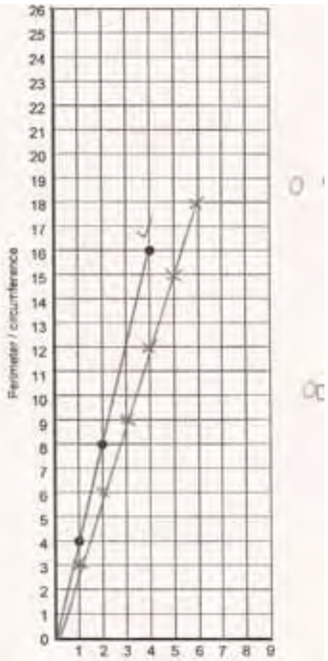
16 X

Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?

6' X

Mark a point for this square on the graph.



(a) Explain why the line passes through the point (0, 0).

because it always connect to 0 X

(b) Explain why the line is straight.

because it goes in the same pattern X

4. Draw a circle around the correct equation of the straight line on the graph.

$x + y = 4$ $\quad y = x + 4 \quad y = 4x \quad y = \frac{x}{4}$

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference. Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same:

There not the same X

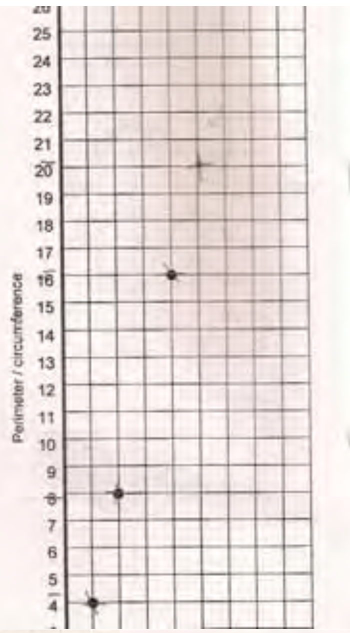
Different:

the difference is That There etier close or far apart. X

9

Student J

The points on this graph show the perimeters of squares of different sizes.
For example, a square with sides 2 inches long has a perimeter of 8 inches.



1. What is the perimeter of a square with sides 5 inches long?
   P=20

   Mark a point for this square on the graph.

2. How long are the sides of a square with perimeter 12 inches?
   P=48

   Mark a point for this square on the graph.

(a) Explain why the line passes through the point (0, 0).
To make it easy to find out what it is.

(b) Explain why the line is straight.
So you can understand it better.

4. Draw a circle around the correct equation of the straight line on the graph.

$$x + y = 4 \qquad y = x + 4 \qquad y = 4x \qquad y = \frac{x}{4}$$

This table shows the circumferences of circles with different diameters.
The circumferences have been rounded to one decimal place.

Diameter of circle in inches	1	2	3	4	5	6
Circumference in inches	3.1	6.3	9.4	12.6	15.7	18.8

5. For each circle, mark an (X) on the graph to show its diameter and circumference.
   Join the Xs with a straight line.

6. Write down one thing that is the same and one thing that is different about the line for the squares and the line for the circles.

Same: You find out what it is.

Different: line for squares are easy [illegible] harder

Eighth Grade

8th Grade Task 2 Squares and Circles

Student Task	Work with perimeter and circumference of squares and circles. Use and interpret line graphs and their equations.
Core Idea 3 Algebra and Functions	**Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.** • Identify functions as linear or nonlinear, and contrast their properties from tables, graphs, or equations. • Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

Based on teacher observation, this is what eighth graders knew and were able to do:
- Find perimeter of a square given the side length
- Graph the points
- Find the side length of a square given the perimeter

Areas of difficulty for eighth graders:
- Understanding slope or rate of change
- Connecting the origin to the context of the perimeter
- Using mathematical vocabulary to describe patterns or trends
- Understanding the "continuousness" of a line
- Explaining why a line is straight
- Comparing and contrasting linear functions, picking mathematically significant details of the graphs

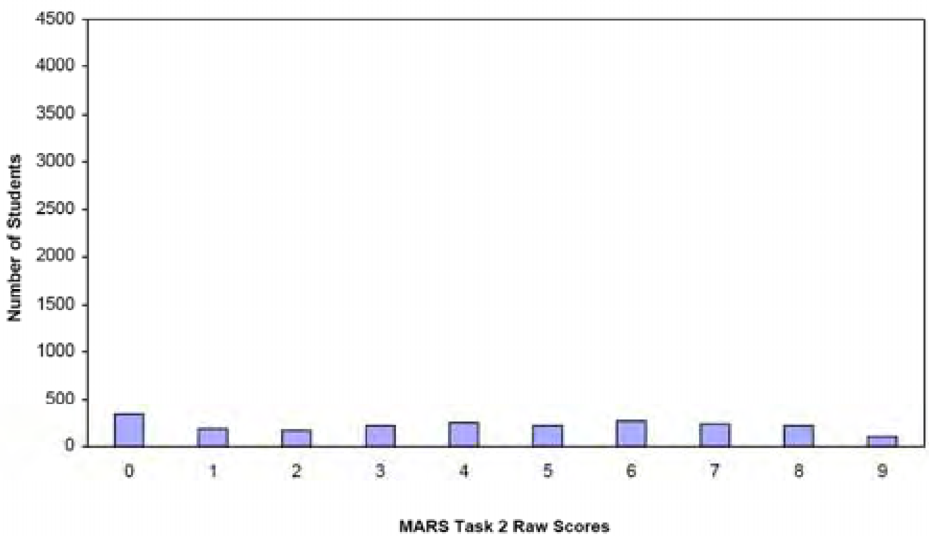
Task 2 - Squares and Circles

Mean: 4.21 StdDev: 2.82

Table 41: Frequency Distribution of MARS Test Task 2, Grade 8

Task 2 Scores	Student Count	% at or below	% at or above
0	332	14.9%	100.0%
1	191	23.4%	85.1%
2	175	31.2%	76.6%
3	221	41.1%	68.8%
4	248	52.2%	58.9%
5	219	62.0%	47.8%
6	271	74.1%	38.0%
7	241	84.9%	25.9%
8	227	95.1%	15.1%
9	110	100.0%	4.9%

Figure 50: Bar Graph of MARS Test Task 2 Raw Scores, Grade 8



The maximum score available on this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Many students, 76%, were able to calculate the perimeter for a square with a side of 5", plot the point, and draw a line through all the points on the graph. More than half the students, 59%, were able to find the perimeter, graph point one, connect the line, choose the equation for the square, and either find the side length or state what was the same between the square and circle graphs. About 1/4 of the students could find the perimeter and side length, draw the lines for squares and circles, explain why the line for squares was the same as circles, choose an equation to represent the squares, and state something that was the same about the square and circle graphs. About 5% of the students could meet all the demands of the task, including explaining why the graphs for squares went through the origin and finding a significant feature that was different between the square and circle graph. 15% of the students scored no points on this task. 90% of them attempted the task.

Squares and Circles

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	Almost 6% of the students calculated the perimeter, but failed to put in on the graph or didn't locate the point correctly. 5% of the students thought the perimeter was 5. 6% of the students did not attempt to connect the points on the graph. 5% made zigzagging lines instead of straight lines.
2	Students were able to calculate the perimeter for a square with a side of 5", plot the point, and draw a line through all the points on the graph.	13% of the students calculated perimeter of a square with a side of 12 instead of finding the side length of a square with a perimeter of 12. 6% found the side length but did not or could not plot the point.
4	Students were able to find the perimeter, graph point one, connect the line, choose the equation for the square, and either find the side length or state what was the same between the square and circle graphs.	10% of the students explained that the line was straight because the points lined up. 7% thought the line was straight because it was a pattern. 17% of the students thought the equation for side length and perimeter of a square was $y = x + 4$, 9% picked each of the other incorrect choices.
7	Students could find the perimeter and side length, draw the lines for squares and circles, explain why the line for squares was the same as the line for circles, choose an equation to represent the squares, and state something that was the same about the square and circle graphs.	Students could not explain why the line for perimeter goes through the origin. About 30% think all lines go through the origin. Students had trouble explaining what was different about the two graphs. 10% thought the difference was whole numbers and decimals.
9	Students understood perimeter and side length and could explain why it would pass through the origin as well as why the graph was linear. Students could plot decimal values and compare and contrast two graphs using significant features, like origin and slope.	

Implications for Instruction

Students need frequent opportunities to relate equations to a familiar context, so that they can make conclusions about how the graph, equation, and context fit together. They need to think about what in the context causes the graph to go through the origin. Why does that make sense? When do graphs not go through (0,0)? What about the situation causes some other y-intercept to make sense?

Students need to have opportunities to compare and contrast graphs of different functions. Through rich classroom discussion they develop insights into the important features of graphs and why they are relevant. They should be comfortable with important properties of graphs: positive or negative slope, degree of steepness or rate of change, and y-intercept.

Some students had difficulty identifying the graph showing the relationship between the side length and perimeter of a square. They could usually state that the value was increasing by 4's. However many students saw this is additive, $y= x + 4$, rather than multiplicative, $y = 4x$. They do not see the relationship between "equal groups" and multiplication.

Some students had difficulty graphing decimal values. Others had difficulty thinking about how rounding might have effected the values in the table for circumference. Students need to work with a variety of functions, which should include those where numbers are not always whole numbers and where rounding might be helpful to develop a more robust understanding of rounding and growth rates.

Ideas for Action Research

Looking at Classes of Graphs: Starting point for generalizations

Design an exploration about comparing and contrasting graphs. Give students a series of graphs, such as: $y = x + 4$, $y = 4x$, $y = 4x + 4$, $y = x – 4$, $y = x/4$, $y = 4 – x$

Ask students in pairs to think of situations that could fit each equation. Have groups share out their situations and be prepared to prove why that situation matches the equations. Try to get them to discuss why in each situation the "x" represents a variable. What does that mean in the specific context?

Now ask students to make a table of values and to plot the graphs. Have them write a summary statement about patterns in the tables. Then ask them to compare the graphs for the various equations: How they are alike and how are they different? The groups should make posters to show their findings. Students should start to think about slope and y-intercept as they compare graphs. During discussion, try to push them to explain why tables that go up by 4 are not the same as the equation $y = x + 4$. Can they give of reason for deciding which graph might be the steepest? Really try to let them come up with the ideas. Try to capture some of these ideas on a poster for later reference.

Now ask them to predict what a graph for a different equation might look like and give reasons for their prediction. New equations might be: $y = 3x$, $y = x + 5$, $y = x - 7$, $y = 3x + 9$. Then have the students graph the equations and compare the graphs to their predications.

Why is having an investigation around graphing different that giving a lesson on the equation for a line? What things do you think students were picking up from this activity that are different from the traditional lesson? What things do you feel are lost?

Performance Assessment Task
Squares and Circles Grade 8
The task challenges a student to demonstrate understanding of the concepts of linear equations. A student must understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change. A student must be able to identify functions as linear or nonlinear, and contrast their properties from tables, graphs, and equations. A student must explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope. A student must be able to make sense of the difference between a proportional linear equation and one with a constant.
Common Core State Standards Math - Content Standards
<u>**Expressions and Equations**</u> **Understand the connections between proportional relationships, lines, and linear equations.** 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.7 Solve linear equations in one variable. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
Common Core State Standards Math – Standards of Mathematical Practice
MP.7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1), (x–1)(x2+x+1), and (x–1)(x3 +x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
8	2006	9	4	59%