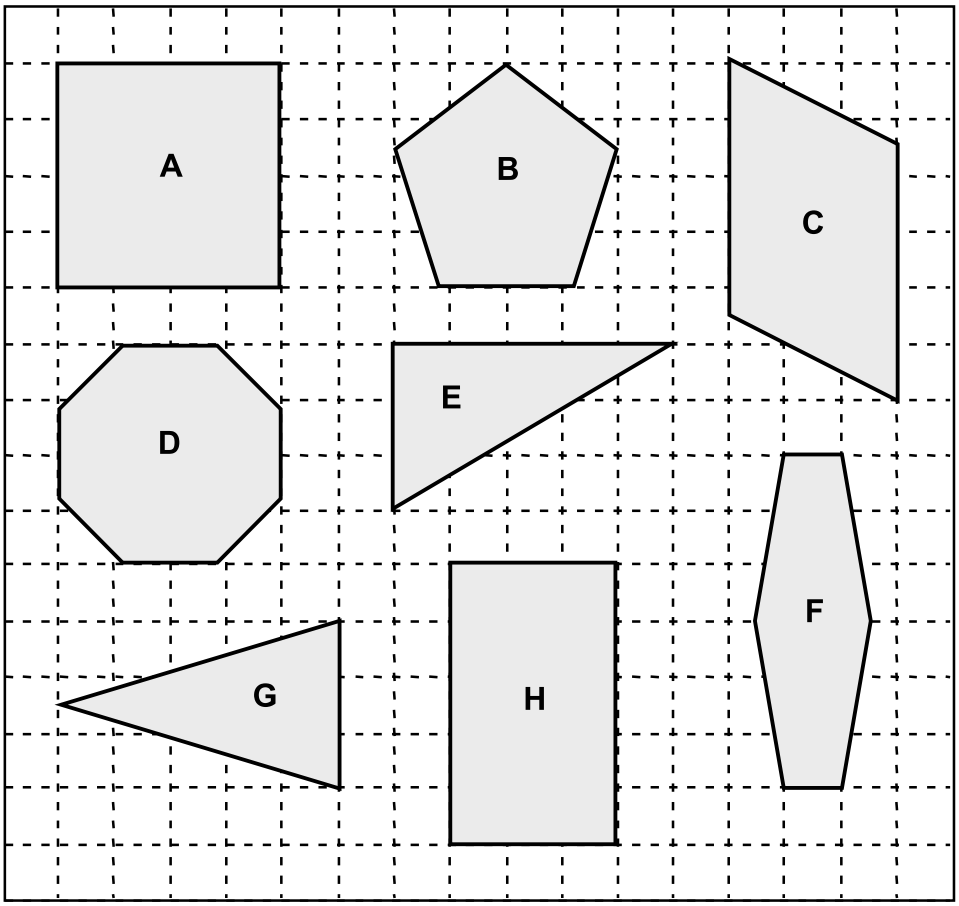
Sorting Shapes

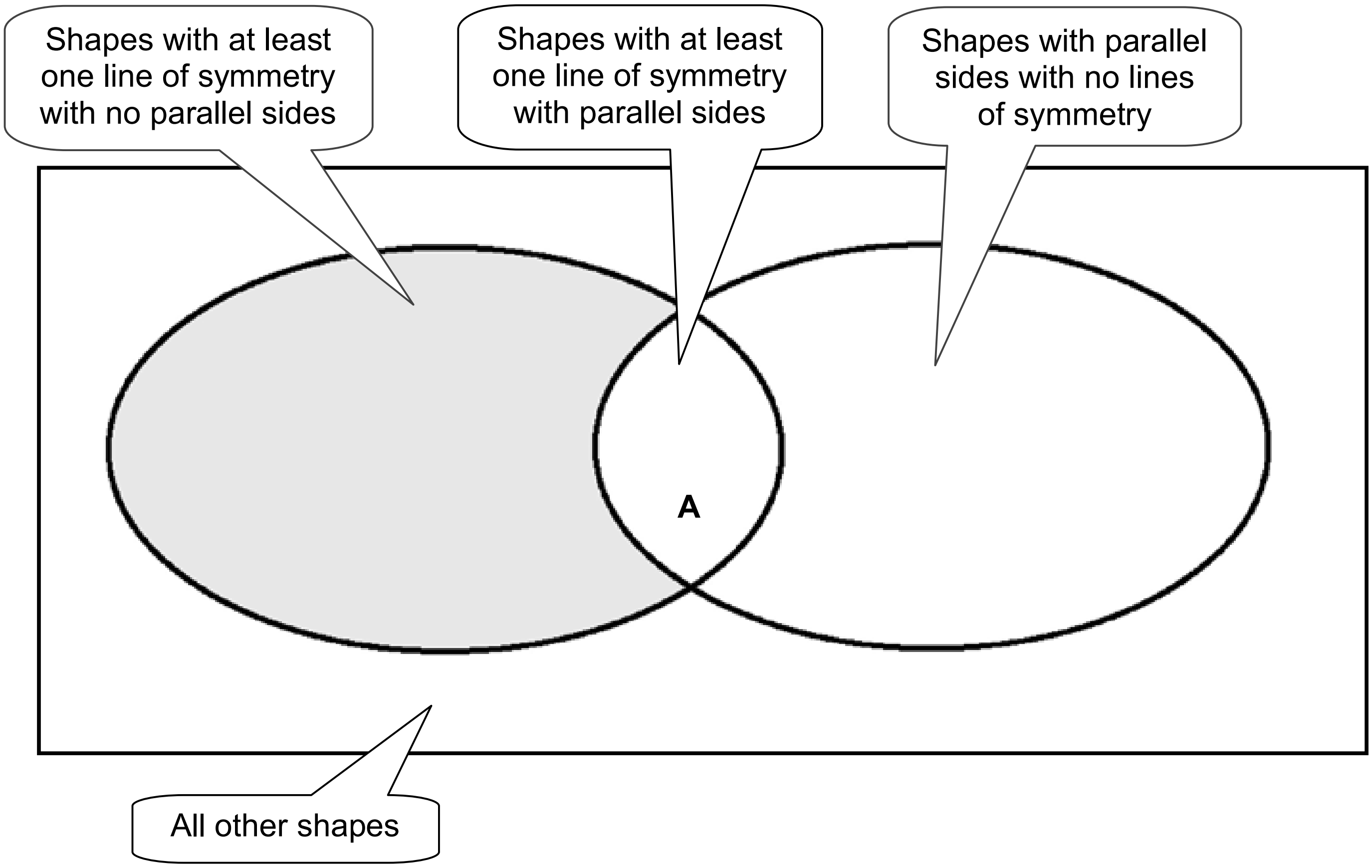
This problem gives you the chance to:
• recognize and name shapes and their properties
• draw a shape to meet given conditions

Here are some two dimensional shapes drawn on square grid paper.

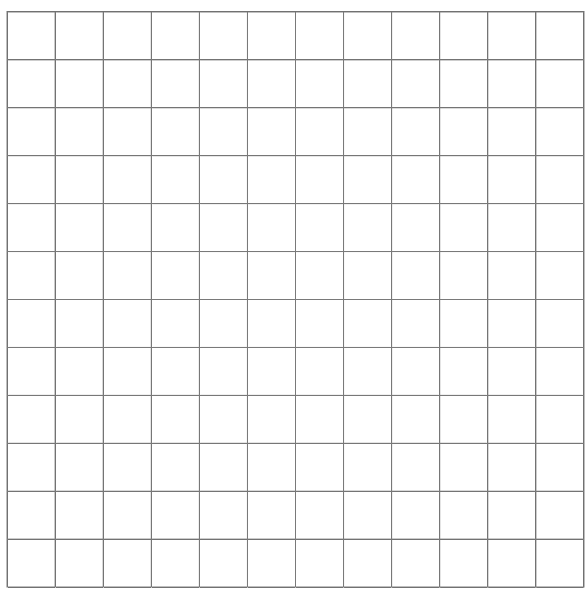


1. What is the mathematical name of shape F? ____________________________
2. How many lines of symmetry does shape D have? _____________________
3. Write the letter of each shape in the correct region of the diagram on the next page.

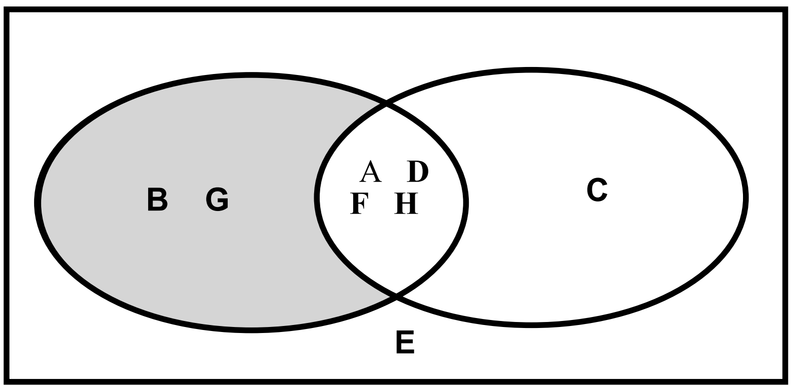
   The first one has been done for you.



4. Draw another shape that could go into the shaded region.



7

Sorting Shapes		Rubric
The core elements of performance required by this task are: • recognize and name shapes and their properties • draw a shape to meet given conditions Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **hexagon**	1	1
2. Gives correct answer: **8**	1	1
3. Completes the diagram.  7 letters correct with no extras *Partial credit* 6 or 5 letters correct 4 or 3 letters correct	3 (2) (1)	3
4. 1 point for a shape with line(s) of symmetry, and 1 point for a shape with no parallel sides.	1 1	2
Total Points		**7**

Sixth Grade – Task 3: Sorting Shapes
Work the task. Look at the rubric. What geometric attributes does a student need to understand to work this task?

What types of work have your students done with attributes?
Look at student work on part one. How many of your students put:

Hexagon	Trapezoid	Parallelogram	3-D shape	Diamond	Pentagon	Octagon	Quadrilateral

Besides memorization, how do we give students meaningful ways to talk about mathematics that would allow them to use geometric terms in context?
How do we help students develop the academic language of mathematics?
Look at student work on part two, lines of symmetry. How many of your students put:

8	4	6	2	Other

Do we give students enough opportunities to think about symmetry that is not on the vertical or horizontal axis?
Do students get opportunities to work with rotations? How might thinking about rotational symmetry help the student find additional lines of symmetry?
Students need to understand the logic of sorting and be able to think about more than one attribute at a time to complete the diagram in part three. Many students had difficulty thinking about a negative attribute like no parallel sides.
How many of your students misplaced:

B	C	D	E	F	G	H

Which shapes or categories seemed most difficult for them?
Have your students done classifying or sorting activities with either number or geometric attributes?
Have they had the opportunity to make their own categories to help them see the logic of the sort?

Being able to design a shape with given attributes is quite challenging and involves synthesizing previous knowledge and apply it in unusual or new ways. Look at student work on part four.

- How many could design a shape matching both constraints?
    - Was their shape a triangle?
    - A circle?
    - Other?
- How many students drew shapes with no lines of symmetry?
- How many students drew shapes with parallel sides?

What opportunities do students have to investigate designing their own shapes to match a given set of attributes or to see if it is possible to have two given sets of attributes? How do these types of activities support the logic needed later for doing geometric proofs and working with more complex geometric properties?

Looking at Student Work on Sorting Shapes

Identifying the name of the shape for F was challenging for students. More than half the students missed this question. Below are common responses for naming shape F:

Rhombus	Quadrilateral	Trapezoid	Pentagon	Diamond	Polygon
6.5%	3%	5%	6.5%	3%	6%

What attributes of F might students have been thinking about when picking these names?

Finding the lines of symmetry for shape D was also difficult for students.

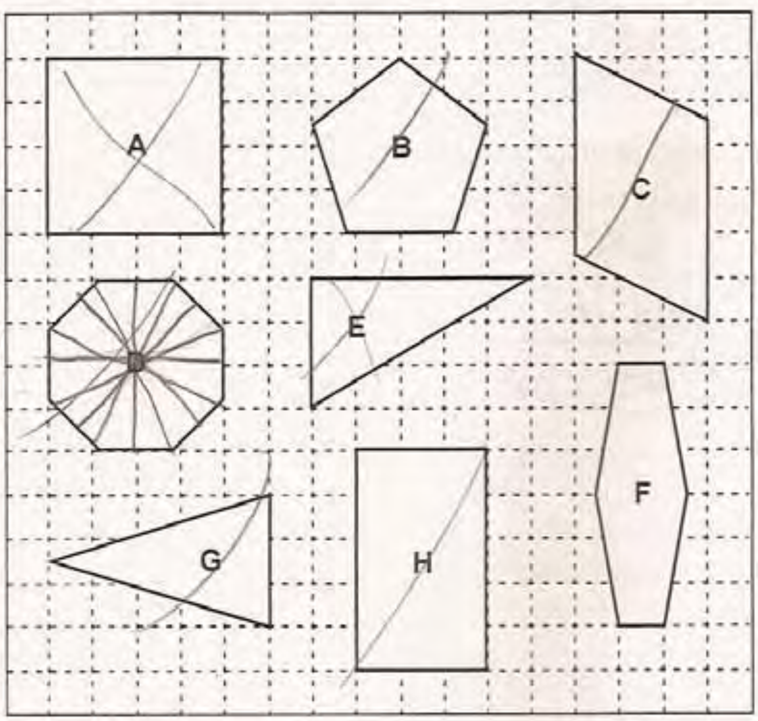
4 lines of symmetry	2 lines of symmetry	6 lines of symmetry
40%	3%	10%

Much of geometry seems to be about opportunity to learn. Students with scores of 9 and 10 for Nuts and Smallest and Largest might score only 1 or 2 points on Sorting Shapes. What are the types of thinking these students are missing?

Student A has a good score with 6 out of 7 points. Notice that the student used the diagram to help make sense of the lines of symmetry. The students didn't see that shape E had no line of symmetry. While the student was able to draw a shape with no parallel lines and a line of symmetry, the student struggles with scale.

Student A

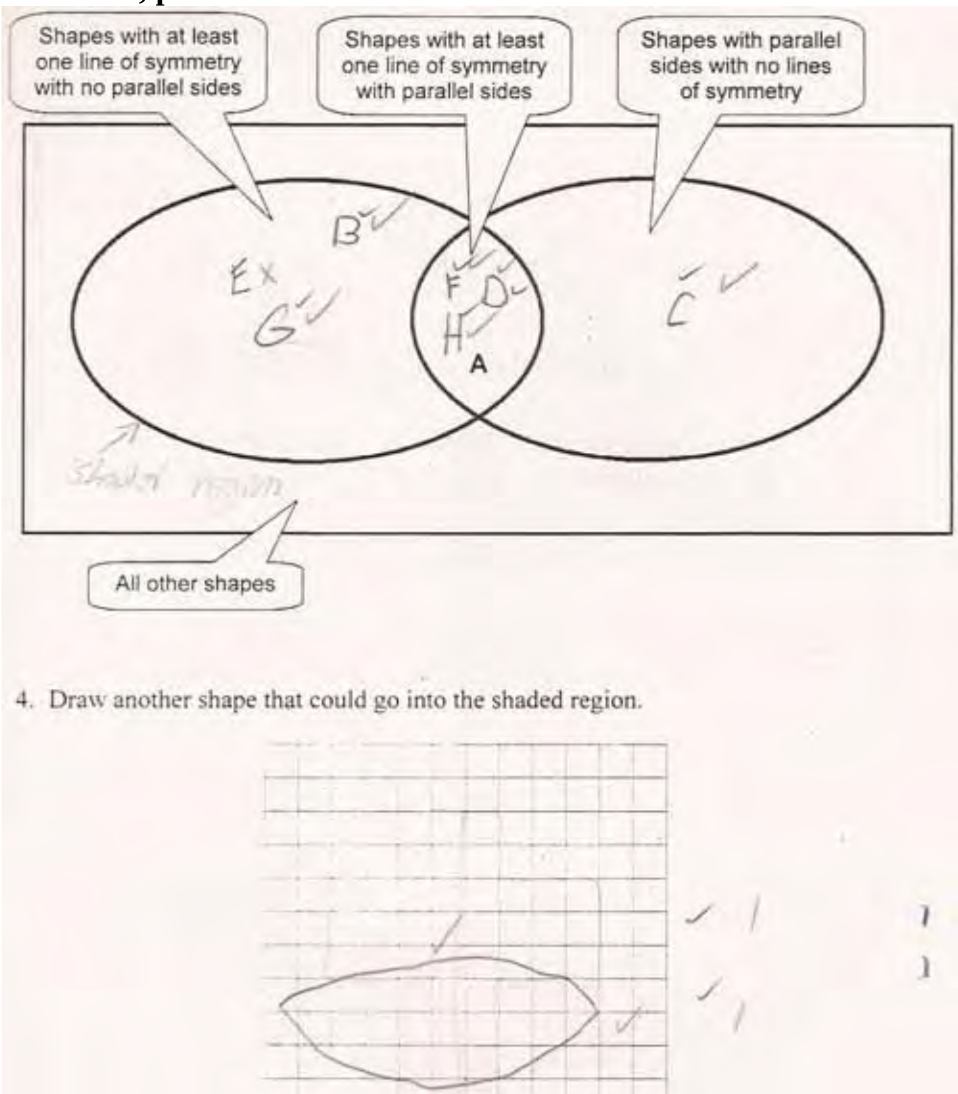
Here are some two dimensional shapes drawn on square grid paper.



1. What is the mathematical name of shape F? hex;gon

2. How many lines of symmetry does shape D have? 8

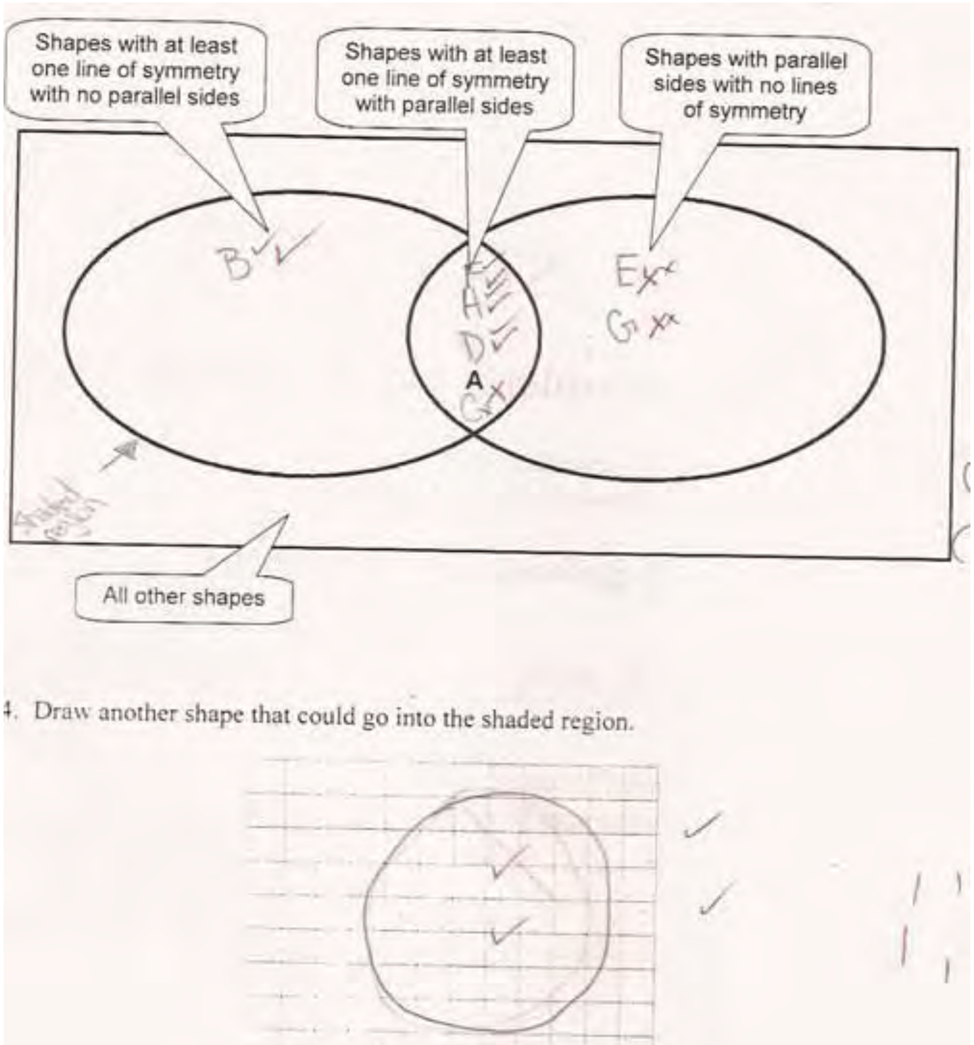
Student A, part 2



4. Draw another shape that could go into the shaded region.

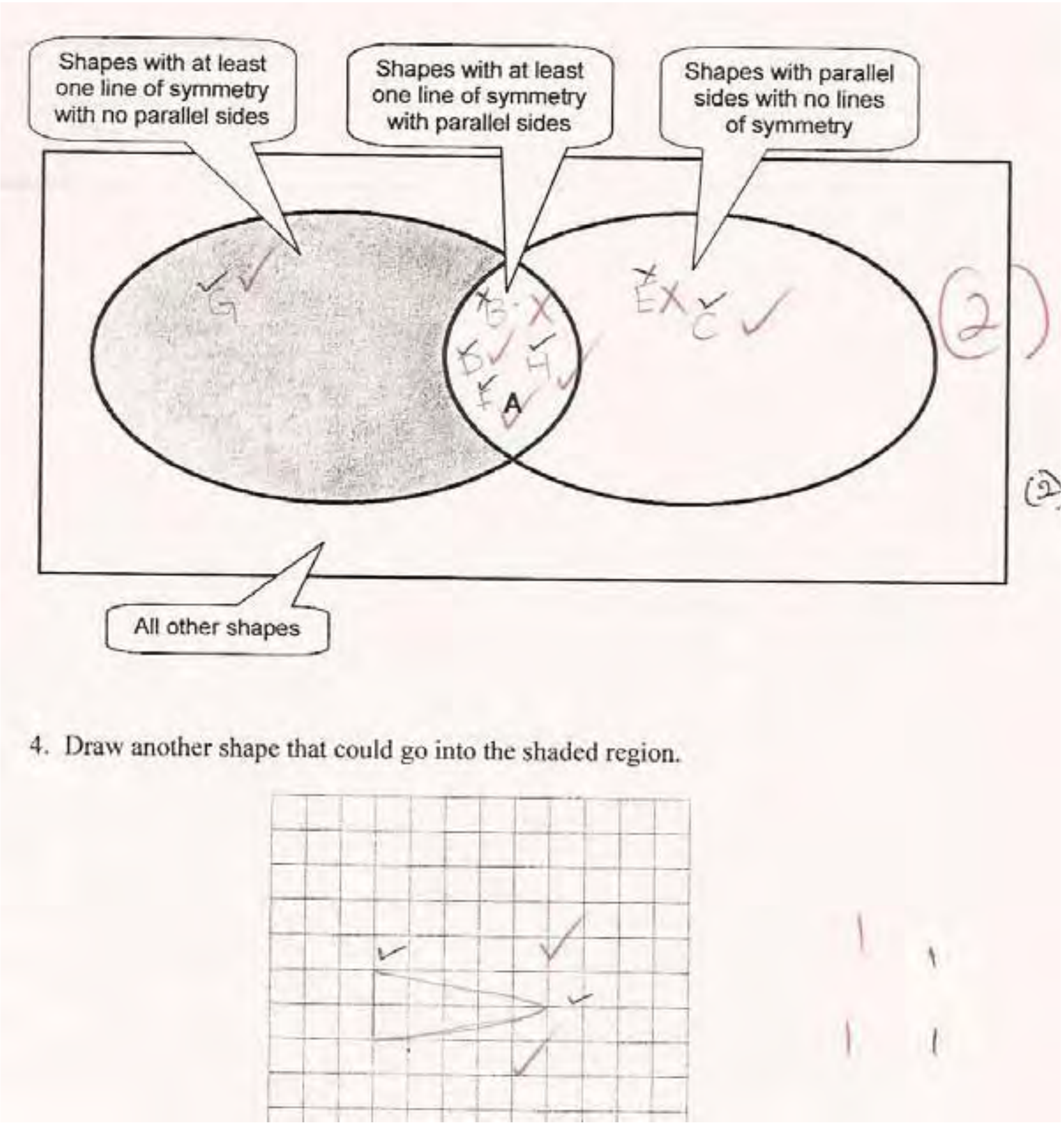
Student B thought there were 6 lines of symmetry. The student did not see that shape C has no line of symmetry or that neither shape E or G have no parallel sides. The student was able to design a shape to meet the constraints of at least one line of symmetry and no parallel sides.

Student B



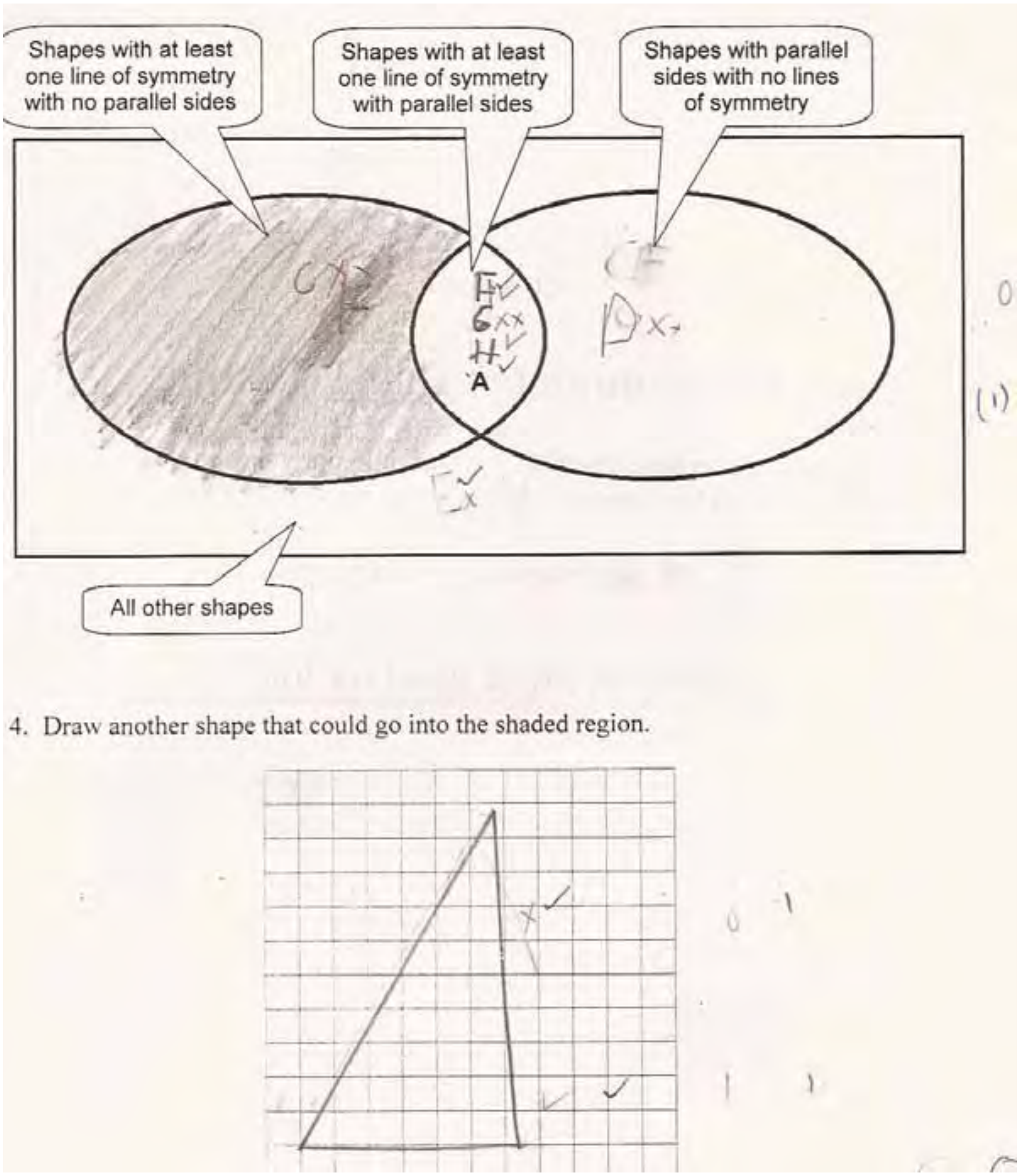
Student C uses the grid lines to help design the shape around a line of symmetry. The student makes the common error of thinking the pentagon has parallel sides and forgetting that the right triangle has no parallel sides.

Student C



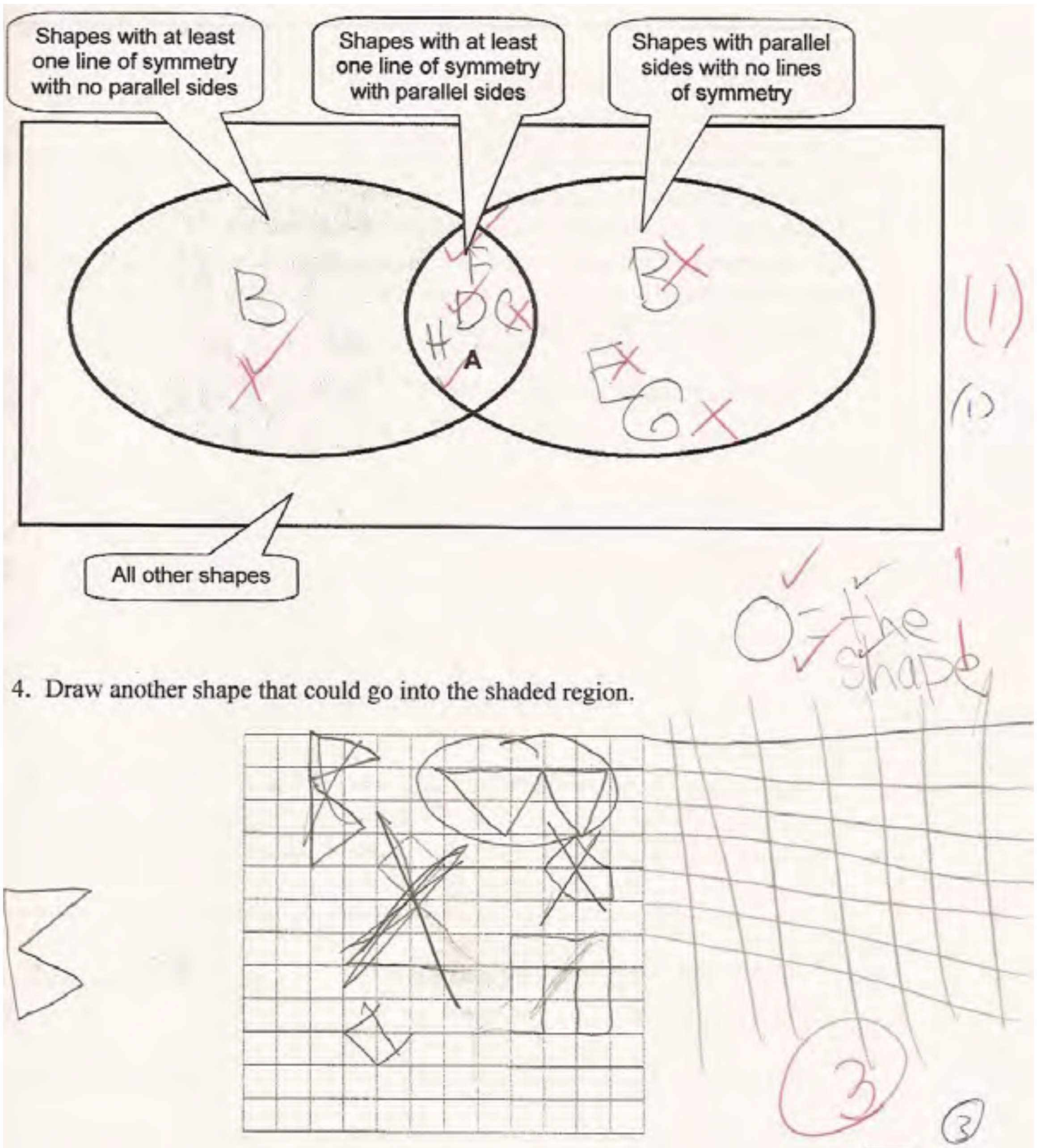
Student D thought the hexagon was a parallelogram. Students don't see that all sides must have a parallel partner or that parallelogram is a type of 4-sided figure. The student fails to see that C, the parallelogram, does not have a line of symmetry; that G, the isosceles triangle, does not have parallel sides; or that D, the octagon, has lines of symmetry. Students have trouble tracking all the constraints.

Student D



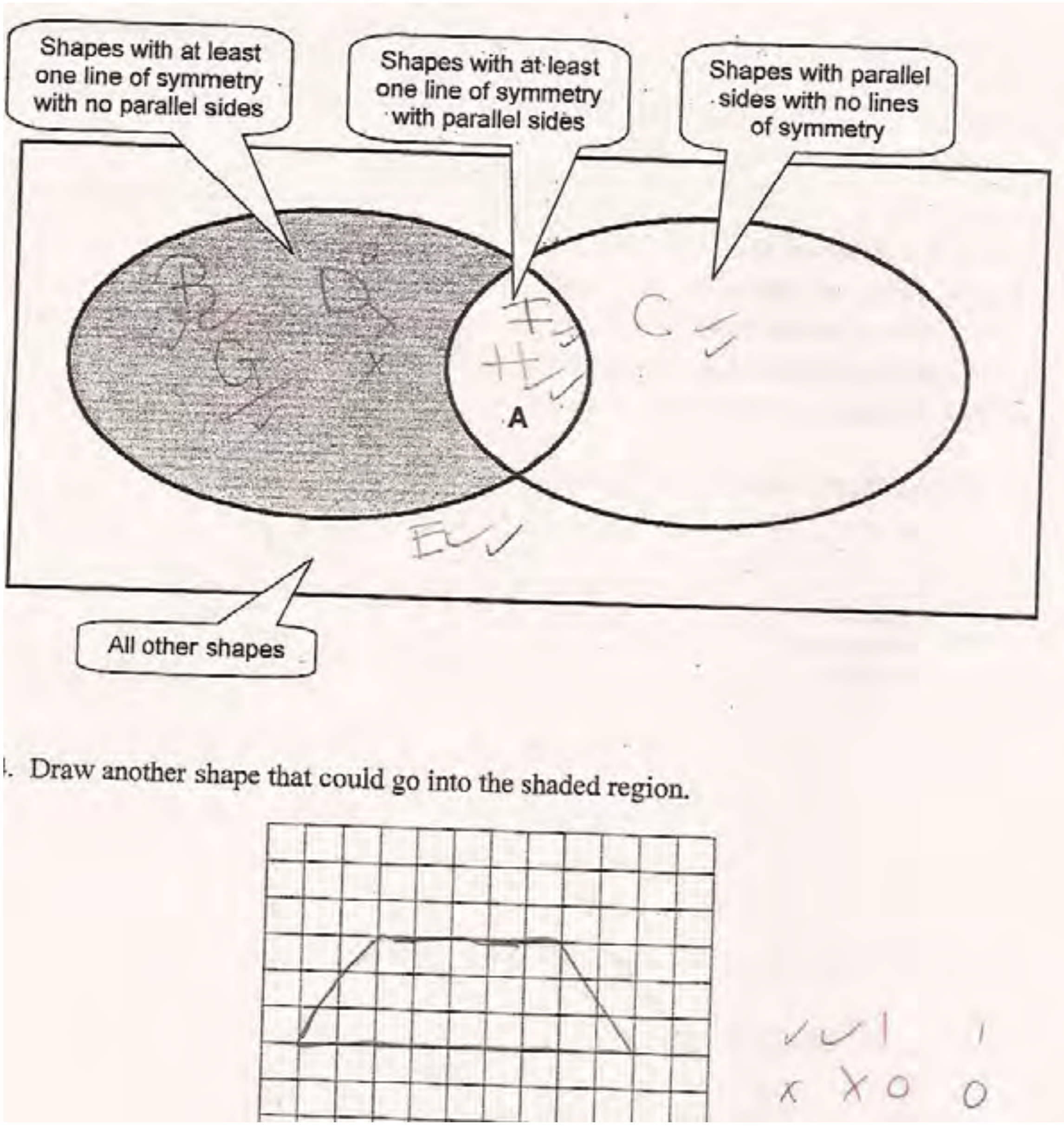
Student E thinks there are only 4 lines of symmetry in the octagon. The student calls the hexagon a polygon. The student has trouble sorting shapes by their attributes. However, with struggle and persistence the student is able to design a shape to meet the constraints in part four.

Student E



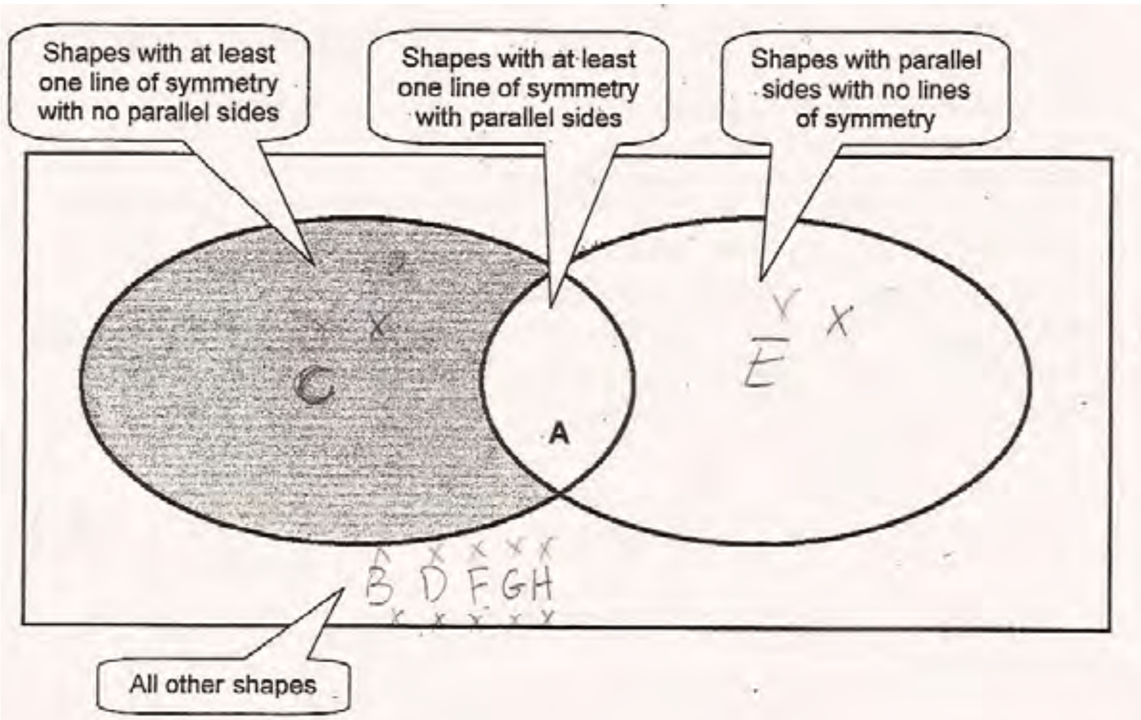
Student G thinks the hexagon, F, is a rhombus. The student thinks shape D has only 2 lines of symmetry. The student also does not see that shape D, the octagon, has parallel sides. Understanding parallel sides is difficult for this student and shows up in the design, which should have been a shape with no parallel sides.

Student G

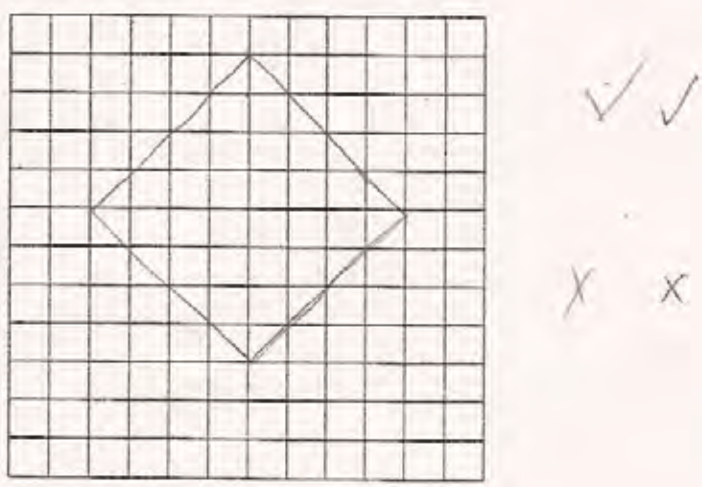


Student H does not seem to understand the logic of sorting. The student seems to think that only 1 letter can go in each section. The student may have trouble seeing attributes. The student thought that the hexagon, F, was a diamond.

Student H



4. Draw another shape that could go into the shaded region.



Sixth Grade

6^{th} Grade Task 3 Sorting Shapes

Student Task	Recognize and name shapes and their properties and draw a shape to meet given conditions.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes, develop mathematical arguments about geometric relationships; use symmetry.** • Describe, classify, and understand relationships among types of two-dimensional objects, using their defining properties. • Understand line symmetry.

Mathematics in the task:
- Ability to look at geometric shapes, recognize key attributes, and use those attributes to name shapes
- Ability to count lines of symmetry
- Ability to use the logic of classifying and sorting to place geometric shapes according to constraints about their attributes
- Ability to design a shape to meet given constraints about the geometric attributes

Based on teacher observations, this is what sixth grade students knew and were able to do:
- Identify a hexagon
- Place the rectangle and parallelogram on the diagram based on the attributes

Areas of difficulty for sixth graders:
- Understanding the logic of classifying shapes
- Thinking about negative attributes, e.g. no parallel lines
- Using attributes to design a shape to meet given constraints

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 6

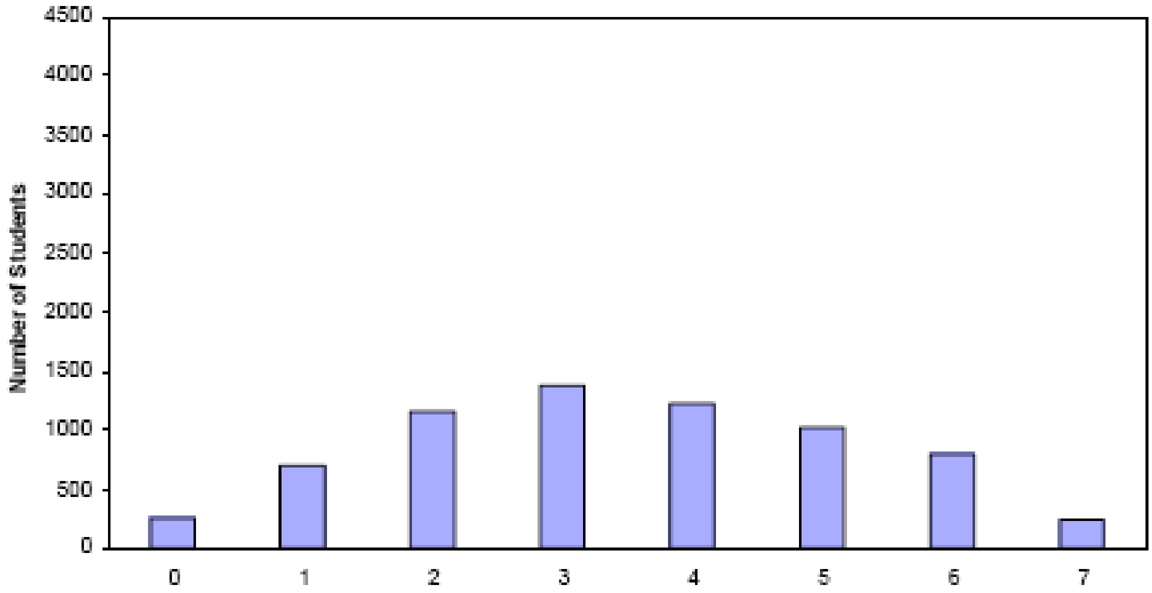
Task 3 – Sorting Shapes

Mean: 3.49 StdDev: 1.76

Table 32: Frequency Distribution of MARS Test Task 3, Grade 6

Task 3 Scores	Student Count	% at or below	% at or above
0	259	3.8%	100.0%
1	696	14.1%	96.2%
2	1161	31.2%	85.9%
3	1379	51.5%	68.8%
4	1230	69.6%	48.5%
5	1013	84.5%	30.4%
6	804	96.4%	15.5%
7	246	100.0%	3.6%

Figure 41: Bar Graph of MARS Test Task 3 Raw Scores, Grade 6



The maximum score available for this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 96%, could draw a shape with a line of symmetry or see that the octagon has 8 lines of symmetry. Many students, 69%, could draw a shape with a line of symmetry, name the hexagon and correctly locate at least 3 shapes in the diagram or draw a shape with both a line of symmetry and no parallel sides and locate at least 3 shapes in the diagram. Almost half of the students, 49%, could name the hexagon, find 3 shapes in the diagram, and design a shape to meet the constraints. Less than 5% of the students could meet all the demands of the task including finding the lines of symmetry of an octagon and locating all the shapes in a Venn diagram of attributes. Less than 5% of the students scored no points on this task. 80% of the students with this score attempted the task.

Sorting Shapes

Points	Understandings	Misunderstandings
0	80% of the students attempted the task.	Students had difficulty designing a shape with one line of symmetry. Many tried to use a right triangle.
1	Students could draw a shape with at least one line of symmetry or find the number of lines of symmetry in an octagon.	40% of all students thought the octagon had 4 lines of symmetry. 10% thought it had 6 lines of symmetry. 3% thought it had 2 lines of symmetry.
3	Many students could draw a shape with a line of symmetry, name the hexagon and correctly locate at least 3 shapes in the diagram or draw a shape with both a line of symmetry and no parallel sides and locate at least 3 shapes in the diagram.	Many students had trouble naming the hexagon. Common errors included trapezoid, rhombus, parallelogram, quadrilateral, pentagon, diamond, and polygon.
4	Almost half of the students could name the hexagon, find 3 shapes in the diagram, and design a shape to meet the constraints.	Students had difficulty thinking about two attributes at the same time. Students had more difficulty with negative attributes: no line of symmetry or no parallel sides.
6		Students still could not find the number of lines of symmetry for the octagon.
7	Students could use attributes of geometric shapes for naming, classifying and sorting shapes. They could think about, count, and design shapes with lines of symmetry and no parallel sides. Students understood the logic of sorting shapes by more than one attribute.	

Sorting Shapes
Reference Note:

From "The van Hiele Levels of geometric Understanding" by Marguetie Mason, as found on the website: www.mcdougallittel.com/stuate/tx/corr/levels

Level 1 (Visualization) : Student recognize figures by appearance alone, often by comparing them to a know prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.
Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

The van Hiele theory indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection. Research indicates that the levels are sequential to arrive an any level above 0, students must move through all prior levels without skipping.
If instruction is given at a level of thought that is above that of the student, the student will, generally, not understand the content that is being taught. Usually, the student will try to memorize the material and may appear to have mastered it, but the student will not actually understand the material. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation.

Most high school geometry teachers think at the fourth or fifth van Hiele level. Research indicates that most students starting a high school geometry course think at the first or second level. The teacher needs to remember that although the teacher and the student may both use the same word, they may interpret it quite differently. For example, if a student is at the first level, the word "square" brings to mind a shape that looks like a square, but little else. At the second level the student thinks in terms of properties of a square, but may not know which ones are necessary or sufficient to determine a square. The student may feel that in order to prove that a figure is a square, all the properties must be proved. The teacher, who is thinking at a higher level, knows not only the properties of a square, but also which ones can be used to prove that a figure is a square.

Implications for Instruction:

Students need practice with sorting and categorizing geometric shapes around a variety of attributes. This practice should include sometimes making them decide on the attributes, which would best sort the group. This helps students develop logical reasoning and increase spatial visualization needed for later work in geometry. Students also need to have practice designing shapes to meet certain constraints. Sometimes the challenge should include things that can't be made. Students found it easier to design a shape with a given attribute, like has at least one line of symmetry, than to design a shape that did not have an attribute, like no parallel sides. Thinking about a negative property was difficult for students both in the sorting and designing phase of the task.

Students need opportunities to talk about geometric figures and what they see. Students need to have opportunities to draw and model with geometric objects and describe important properties of the shapes. Students need to exchange ideas about their observations and discoveries, making their learning or understanding explicit. Students also need experiences for solving open-ended tasks involving investigations.

Students working at the visualization level need activities like:

- sorting, identifying, and describing shapes
- manipulating physical models
- seeding different sizes and orientations of the same shape to help differentiate between relevant and irrelevant features
- building, drawing, making, putting together, and taking apart shapes

Students working at the analysis stage need activities like:

- Shifting from simple identification to properties, by using concrete or virtual models to define, measure, observe, and change properties
- Using models and/or technology to focus on defining properties, making property lists, and discussing sufficient conditions to define a shape
- Doing problem solving, including tasks in which properties of shapes are important components
- Classifying using properties of shapes

Students working at abstraction or informal deduction are ready for activities like:

- Using informal, deductive language (all, some, none, if-then, what if, etc.)
- Using models drawings as tools to look for generalizations and counter-examples
- Making and testing hypotheses
- Using properties to define a shape or determine if a particular shape is included in a given set

Ideas are taken from http://images.rbs.org/cognitive/van hiele.shtml

For detailed lessons plans see: www.pen.k12.va.us/VDOE/Instruction/Elem_M/midgeo1.pdf

Performance Assessment Task
Sorting Shapes Grade 6 task aligns in part to CCSSM grade 5
This task challenges a student's knowledge of 2-dimensional geometrical shapes to sort shapes and to design a shape that has a given set of attributes. A student must be able to look at number of sides or angles, lines of symmetry, and parallel sides to sort shapes. A student must understand the logic of sorting to see where shapes with multiple attributes will fall within a Venn diagram.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Classify two-dimensional figures into categories based on their properties.** 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
Grade 6	2006	7	4	49%