Rhombuses

This problem gives you the chance to:
- use properties of a rhombus
- use the Pythagorean Rule
- check figures for similarity

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD. P, Q, R and S are the mid-points of the sides of the rectangle.

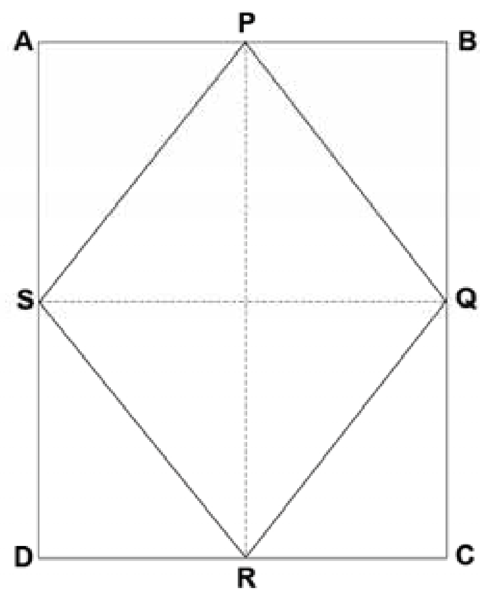
   Explain clearly why PQRS is a rhombus.

   ____

   ____

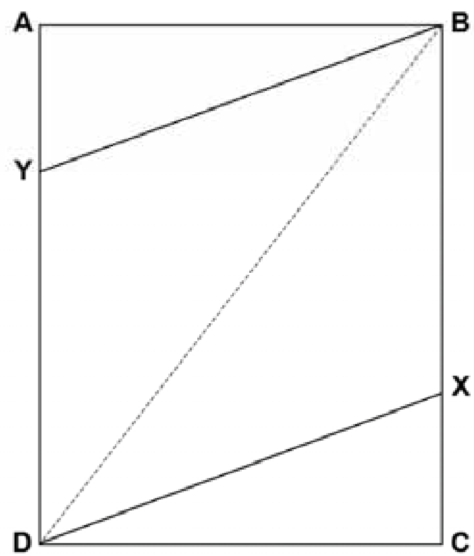
   ____

   ____



2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

   Show that BXDY is a rhombus if AY and CX are 1.8cm.



3. Show that the two rhombuses PQRS and BXDY are similar.

   Explain your reasoning.

   ____

   ____

   ____

   ____

   ____

8

Rhombuses		Rubric
The core elements of performance required by this task are: • use properties of a rhombus • use the Pythagorean Rule • check figures for similarity Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct explanation such as: PQRS is a quadrilateral with four sides of equal measure with explanation. *Partial credit* PQRS is a quadrilateral with four sides of equal measure	 2 (1)	 2
2. Shows that BY = XD = **8.2** cm With correct work such as: $BY^2 = AY^2 + AB^2$ $BY^2 = 1.8^2 + 8^2$ BY = 8.2	1 1	 2
3. In rhombus PQRS, shows that PS = 6.4 In rhombus BXDY, shows that BD = 12.8 Shows that the sides of the triangles PSR and BYD are proportional. Shows that: $\frac{PS}{BY} = \frac{PR}{BD}$ e.g. accept $\frac{6.4}{8.2} = 0.78$ and $\frac{10}{12.8} = 0.78$ Since the sides are proportional, the rhombuses are similar. *Alternatively* Finds angle SPQ = angle YBX = 77.3º Or angle PSR = angle BYD = 102.7º Since the angles are equal, the rhombuses are similar.	1 1 1 1 **or** 2 2	 4
Total Points		**8**

Geometry – Task 5: Rhombuses

Work the task. Examine the rubric.

What concepts, definitions, and procedures did a student need to know to work this task?

Look at student work on part one. How many of your students:

- Talked about the diagonals being perpendicular? Is that enough to define a rhombus? Why or why not?
- How many students did not know the definition of a rhombus? What is your evidence?
- How many students talked about properties unrelated to proving that the shape is a rhombus: all are midpoint, it's a 3-sided figure, opposite sides are congruent, parallel lines, 4 right angles, all the angles are equal, etc.?
- How many students are thinking about looks only: its slanted, it looks like a rhombus, the sides look equal?

Look at student work on part two.

- How many students mislabeled the diagram (put 10 instead of 8.2 for the side of the parallelogram)?
- How many students make assumptions about the angles without proving their size or assumptions about the angles of the bisectors?
- How many students think the rhombus only has 1 diagonal?

Look at student work on part three.

How many students thought that the rhombuses were similar because all rhombuses have equal sides or are proportional?

How many students found the size of side BD?

How many students knew that you needed to find the ratio of the diagonals and compare it to the ratio of corresponding sides?

How many students attempted to prove it by using trig functions to find the corresponding angles?

How many students tried to make justifications that didn't deal with properties of a rhombus: both shapes are inscribed in a rectangle, both diagonals are congruent, they're the same shape and size?

Looking at Student Work on Rhombuses

Student A gives a clear definition for a rhombus in part one, along with a proof of why the sides are equal. Student A is also able to prove that the two rhombuses are similar, by using trig functions to prove the corresponding angles are the same.

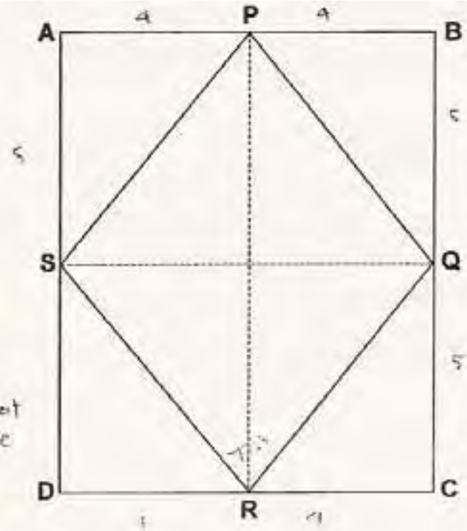
Student A

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD. P, Q, R and S are the mid-points of the sides of the rectangle.

   Explain clearly why PQRS is a rhombus.

   because P, Q, R, & S are midpts, they divide their sides into 2 ≅ sides. ∠A, B, C, & D are rt. ∠s because it's a rectangle. using the pythagorean theorem you can find that PS, PQ, QR, & RS all = $\sqrt{41}$. since this figure has 4 ≅ sides, it is a rhombus.

   $\sqrt{4^2+5^2} = \sqrt{41}$

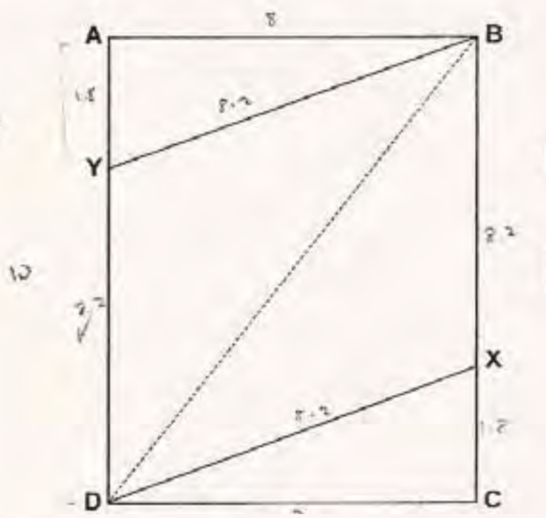


2

2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

   Show that BXDY is a rhombus if AY and CX are 1.8cm.

   $\overline{BY} = \sqrt{8^2+1.8^2} = 8.2$ ✓
   $\overline{XD} = \sqrt{8^2+1.8^2} = 8.2$



3. Show that the two rhombuses PQRS and BXDY are similar.

   Explain your reasoning.

   ∠QRC = $\tan^{-1}\frac{5}{4}$ = 51.34 ∠QRC + ∠SRD = 51.34 · 2 = 102.68

   ∠QRS = 180 − (∠QRC + ∠SRD) = 180 − 102.68 = 77.32 ✓

   ∠XDC = $\cos^{-1}\frac{8}{8.2}$ = 12.68 ∠YDX = 90 − ∠XDC = 90 − 12.68 = 77.32 ✓ 2

   since we know that ∠QRC ≅ ∠XDC, we know that ✓ 2

Student B is able to correctly answer part one and two of the task. In part three the student show that the ratio of sides from PQRS and BYDX are all the same. However unless the diagonals are also in the same ratio or the angles are the same, this is not enough to guarantee the rhombuses are similar.

Student B

3. Show that the two rhombuses PQRS and BXDY are similar.

Explain your reasoning.

✓ $6.4/8.2 = 6.4/8.2 = 6.4/8.2 = 6.4/8.2$ All sides of each rhombus are congruent. They produce the same ratio when divided. If all the corresponding sides of two polygons are proportional, the two polygons are similiar

1

0

1

0

It is unclear if Student C could give a definition of a rhombus. In part one and three the student seems to imply that only opposite sides need to be equal. In fact the diagram in part two shows the rhombus labeled with different size sides. However in part one the student shows in the diagram that all 4 sides are equal and uses the theorem that if the diagonals are perpendicular then the shape is a rhombus.

Student C

Rhombuses

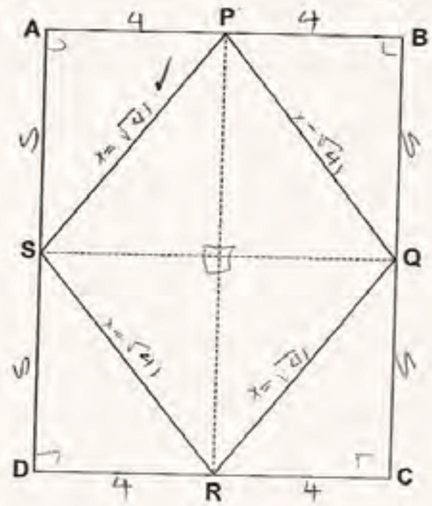
This problem gives you the chance to:
- use properties of a rhombus
- use the Pythagorean Rule
- check figures for similarity

$x^2 = 5^2 + 4^2$
$x = \sqrt{41}$ ✓

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
P, Q, R and S are the mid-points of the sides of the rectangle.

Explain clearly why PQRS is a rhombus.

opposite sides are ≅.
The diagonals form 90° angles.
to perpendicular to each other.
Defn. of rhombus

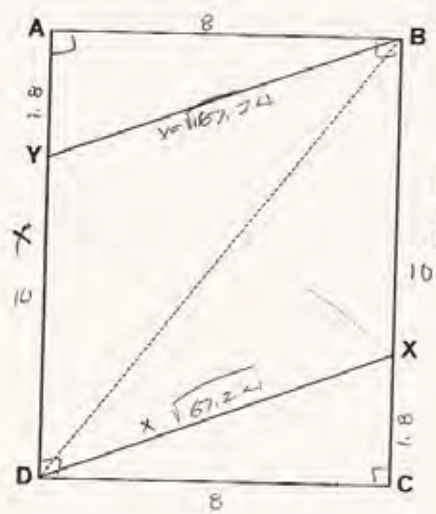


2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

Show that BXDY is a rhombus if AY and CX are 1.8cm.

$x = 8^2 + 1.8^2$ ✓
$x = \sqrt{67.24}$

opposite sides are ≅.



1
0

3. Show that the two rhombuses PQRS and BXDY are similar.
Explain your reasoning.

The opposite sides are ≅.
The opposite angles are ≅.

1
0
0
0

Student D uses conflicting pieces of information. The student states that all 4 sides are equal because of Pythagorean theorem (true). But then goes on to talk about the outside triangles as 3, 4, 5 triangle (not true). The student should see that the hypotenuse isn't be the smallest side. In part three the student uses the definition of similar without quantifying the relationships to prove the shapes are the same. The angles of the diagonals will always be 90 degrees for a rhombus. This does not guarantee similarity.

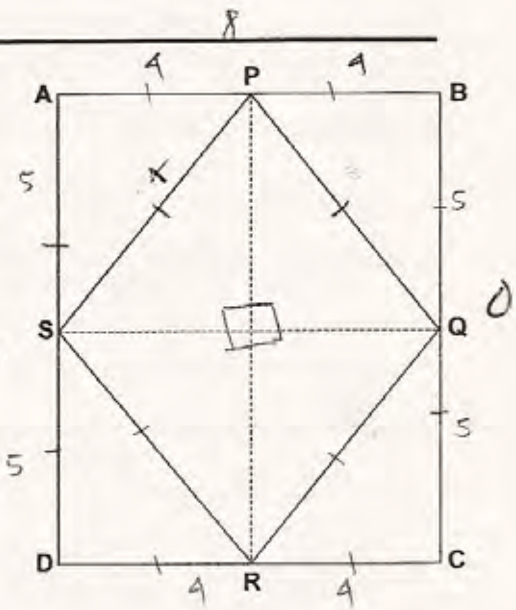
Student D

This problem gives you the chance to:
- use properties of a rhombus
- use the Pythagorean Rule
- check figures for similarity

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
P, Q, R and S are the mid-points of the sides of the rectangle.

Explain clearly why PQRS is a rhombus.

Since the sides of PQRS are all equal using the pythagorean theorem, PQRS is a rhombus. Since the outer Δ's are 3,4 5 Δ's that makes them right triangles, which means that they all have right angles, which makes it a rhombus.



2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

Show that BXDY is a rhombus if AY and CX are 1.8cm.

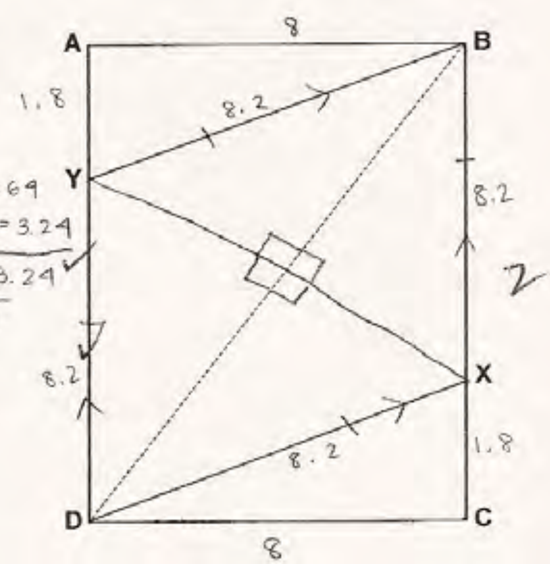
$8^2 = 64$

$1.8^2 = 3.24$

$x = \sqrt{64 + 3.24}$

$x = \sqrt{67.24}$

$x = 8.2$



Student D, continued

3. Show that the two rhombuses PQRS and BXDY are similar.
Explain your reasoning.

PQRS and BXDY are both similar because both are the same shape but different size. Both shapes have congruent sides. The triangles within the rhombus all have right angles

X

0
0
0
0

Students E, F, and G think all rhombuses are similar because rhombuses all have 4 equal sides. *What kind of investigation can you provide for students to help them see why this is not true? Do students have access to Geometer Sketch Pad to help them with the investigation?*

Student E

. Show that the two rhombuses PQRS and BXDY are similar.
Explain your reasoning.

All rhombuses are similar to all other rhombuses. their diagonals make a 90° angle and their all quadrilaterals.

X

0
0

Student F

3. Show that the two rhombuses PQRS and BXDY are similar.
Explain your reasoning.

All the sides of rhombus PQRS and BXDY are proportional because all the sides of a rhombus are congruent

X

0
0
0
0

Student G

Show that the two rhombuses PQRS and BXDY are similar.

Explain your reasoning.

Def. of a rhombus is a parallelogram that has all 4 sides congruent. In addition, both have ⊥ bisectors, and consecutive ∠s are supplementary. 1 0

All rhombuses have similar properties therefore PQRS & BXDY are similar 0

1

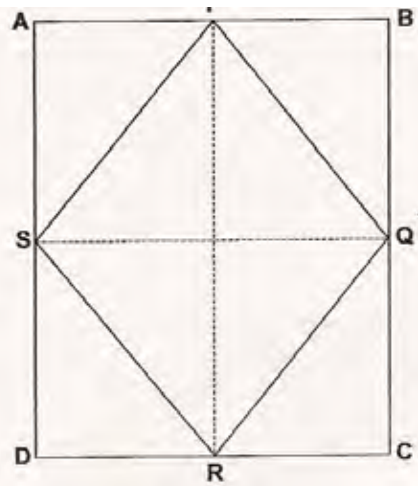
Student H names a lot of properties, but does not have a working definition for rhombus.

Student H

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
   P, Q, R and S are the mid-points of the sides of the rectangle.

   Explain clearly why PQRS is a rhombus.

   · lines are parallel
   · lines are parallel 1

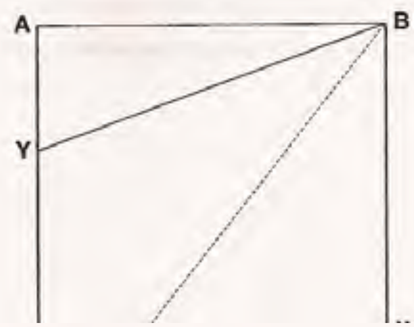


0

2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

   Show that BXDY is a rhombus if AY and CX are 1.8cm.

   It is parallel 1



0

. Show that the two rhombuses PQRS and BXDY are similar.

Explain your reasoning.

· same length
· parallel lines 1
· opposite sides are Equal

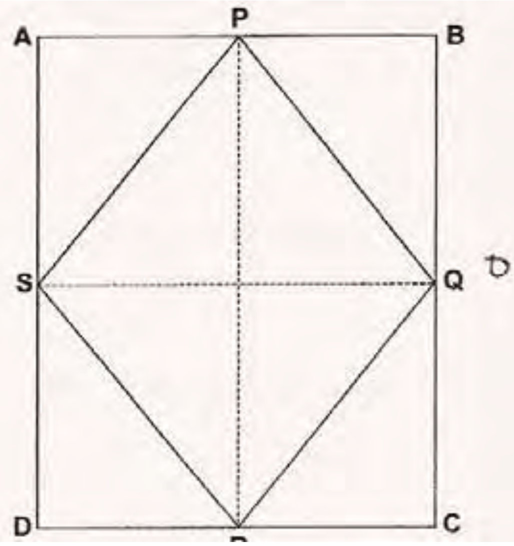
Some students are operating at a level 1 or level 0 of the van Hiele framework. They can't break down the properties of a shape, but think more the overall look of the shape. They are not ready to think about the logic of proofs, because they can't adequately think about the characteristics of the shape. Look at the work of Student I and J.

Student I

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
   P, Q, R and S are the mid-points of the sides of the rectangle.

   Explain clearly why PQRS is a rhombus.

   Because it is a slanted square

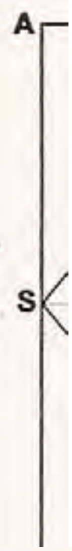


Student J

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
   P, Q, R and S are the mid-points of the sides of the rectangle.

   Explain clearly why PQRS is a rhombus.

   Because ~~they~~ all the sides of PQRS are midpoints of the sides of the rectangle.



Show that the two rhombuses PQRS and BXDY are similar.

Explain your reasoning.

They both have a distance from ~~a~~ ~~at~~ one of the end pts to the other endpt.

Geometry **Task 5** **Rhombuses**

Student task	Use the properties of a rhombus. Use Pythagorean Theorem. Make a justification for similarity.
Core Idea 2 Mathematical Reasoning & proof	Employ forms of mathematical reason and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples.
	• Explain the logic inherent in a solution process.
Core Idea 3 Geometry & Measurement	Analyze characteristics and properties of two and three-dimensional geometric shapes; develop mathematical arguments about geometric relationship; and apply appropriate techniques, tool, and formulas to determine measurements.
	• Understand and use formulas, including solving Pythagorean theorem and trig functions. • Make and test conjectures about geometric objects.

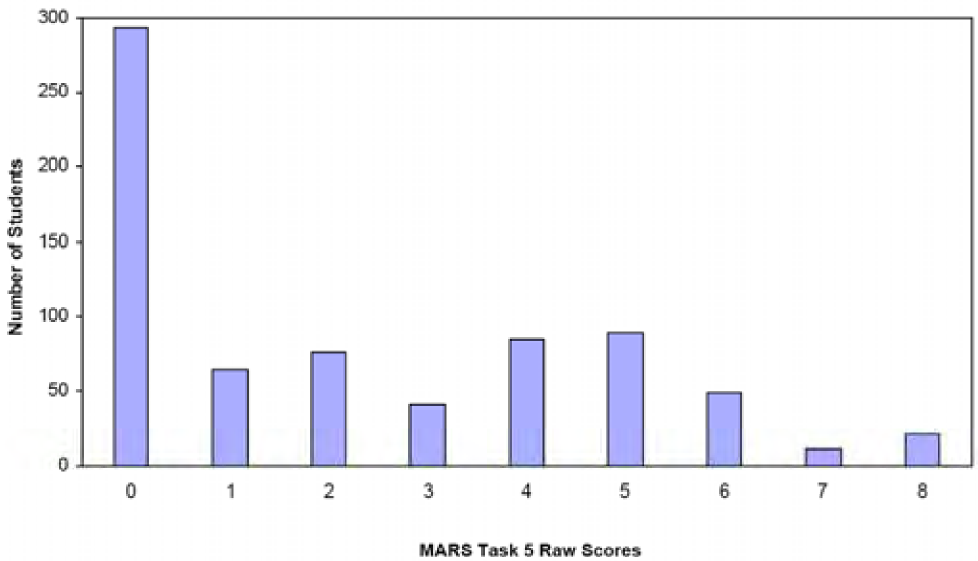
Task 5 - Rhombuses

Mean: 2.28 StdDev: 2.39

Table 54: Frequency Distribution of MARS Test Task 5, Course 2

Task 5 Scores	Student Count	% at or below	% at or above
0	293	40.1%	100.0%
1	65	49.0%	59.9%
2	76	59.4%	51.0%
3	41	65.0%	40.6%
4	85	76.6%	35.0%
5	90	88.9%	23.4%
6	49	95.6%	11.1%
7	11	97.1%	4.4%
8	21	100.0%	2.9%

Figure 63: Bar Graph of MARS Test Task 5 Raw Scores, Course 2



The maximum score available on this task is 8 points.
The minimum score needed for a level 3 response, meeting standards, is 3 points.

More than half, 59%, the students could use subtraction to find the side of the rhombus on the rectangle. About half the students could also use Pythagorean theorem to find the other side of the rhombus to prove that all sides were equal. About 1/3 of the students could also use Pythagorean theorem to find the size of the sides of the rhombus in part one of the task. Slightly more than10% could show that both figures were rhombi. About 3% could meet all the demands of the task including proving that the two rhombi were similar by proving that the angles were equal. 40% of the students scored no points on this task. 85% of the students with this score attempted the task.

Rhombuses

Points	Understandings	Misunderstanding
0	85% of the students with this score attempted the task.	22% of all the students who attempted the task skipped part two. 7% mislabeled the diagram, usually thinking side YD = 10 cm. 2% thought it was a parallelogram and 2% made assumptions about angles without proving those assumptions.
1	Students could use subtraction to find the length of YD in part two.	
2	Students could use Pythagorean theorem to show that all the sides of the inscribed shape in part two were equal proving that the shape was a rhombus.	Students had difficulty showing how shape PQRS in part one was a rhombus. About 8% knew that rhombuses have perpendicular diagonals, but didn't prove that the diagonals were perpendicular. 6% thought that it was a rhombus because the diagonals were congruent. 5% thought it was a rhombus because all the points were midpoints, without clarifying how that related to the definition of a rhombus. 10% said, "it looks like a rhombus."
4	Students could give justifications why the inscribed figures in part one and two were rhombuses by proving that the sides were all equal.	Students did not calculate the side lengths for either rhombus.
5	Students could justify that the figures in part one and two were rhombi and found the length of the side for part 1.	Students had difficulty with proving similarity in part three. 20% of the students thought that all rhombi are similar because all rhombi have equal sides. Students did not know they needed to show that either the angles for both figures were equal or that the diagonals were in the same ratio as the sides. 6% thought they were similar because both figures were inscribed in a rectangle. 4% thought they weren't similar. 12% of the students who did some work on the task did not attempt part three.
6	Students could justify that the figures in part one and two were rhombi and found the length of the side for part 1. They also calculated the ratio of the sides of the two figures.	Students did not realize that they needed to compare the ratios of the diagnonals.
8	Students could justify that the figures in part one and two were rhombi and found the length of the side for part 1. Students in the sample compared the angles rather than comparing the ratio of the diagonals.	

Implications for Instruction

Students seem to be unclear on the working definition for some geometric shapes, like rhombus, based on key attributes like equal sides. Students are also unclear about definition for similarity for rhombi. They don't understand what attributes must be proportional, what attributes must be the same. Why is it enough to show proportionality for the sides of two triangles or two rectangles, but not enough for a rhombus? What makes this shape unique?

Reference Note:

From "The van Hiele Levels of geometric Understanding" by Marguetie Mason, as found on the website: *www.mcdougallittel.com/stuate/tx/corr/levels*

Level 1 (Visualization) : Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.
Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

The van Hiele theory indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection.

If instruction is given at a level of thought that is above that of the student, the student will, generally, not understand the content that is being taught. Usually, the student will try to memorize the material and may appear to have mastered it, but the student will not actually understand the material. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation.

Most high school geometry teachers think at the fourth or fifth van Hiele level. Research indicates that most students starting a high school geometry course think at the first or second level. The teacher needs to remember that although the teacher and the student may both use the same word, they may interpret it quite differently. For example, if a student is at the first level, the word "square" brings to mind a shape that looks like a square, but little else. At the second level the student thinks in terms of properties of a square, but may not know which ones are necessary or sufficient to determine a square.

The student may feel that in order to prove that a figure is a square, all the properties must be proved. The teacher, who is thinking at a higher level, knows not only the properties of a square, but also which ones can be used to prove that a figure is a square.

The article also suggests appropriate teaching strategies for helping students move up through the levels and ways of doing formative assessment for determining the level the student is operation on.

Ideas for Action Research: Investigating quadrilaterals?

Give students an opportunity to investigate the relationships between diagonals and quadrilaterals. What are all the relationships that they can find? What properties will they test? How do they keep track of or organize their information? How can you push them to go beyond finding one or two specific examples to generalizations?

Similar Figures and Proportional Reasoning

A good first definition of similar figures is shapes that "look alike but are different sizes." More precisely, two figures are ***similar*** if all of their corresponding angles are congruent and the corresponding sides are proportional. Many proportional reasoning activities are good connections to geometry. Activities 6.8, 6.9, and 6.10 involve scale drawings and proportional relationships in three-dimensional figures that are similar.

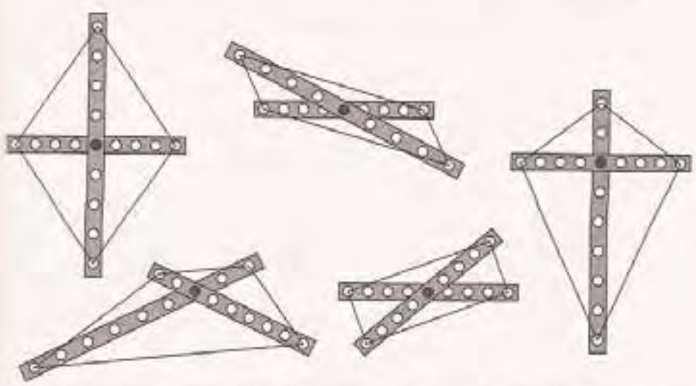


FIGURE 7.13 Diagonals of quadrilaterals.

Quadrilaterals can be determined by their diagonals. Consider the length of each, where they cross, and the angles between them. What conditions will produce parallelograms? Rectangles? Rhombi? Challenge: What properties will produce a nonisosceles trapezoid?

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SHAPES AND PROPERTIES ACTIVITIES

Can they use the materials to find examples of rhombuses that aren't similar? Have them use the overhead to demonstrate to the class?

(This investigation is taken from Teaching Student-Centered Mathematics by John Van de Walle in the chapter helping to develop the van Hiele levels of geometric thought.)

Performance Assessment Task
Rhombuses Grade 10
This task challenges a student to use their knowledge of Pythagorean theorem to show that given rhombuses are similar. A student must use knowledge of proportional sides or angle size to justify that two different rhombuses are similar. A student must construct a convincing argument to show that given 4-sided figures are rhombuses.
Common Core State Standards Math - Content Standards
High School – Geometry – Similarity, Right Triangles, and Trigonometry **Understand similarity in terms of similarity transformations.** G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **Prove theorems involving similarity.** G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
10	2006	8	3	41%