Percent Cards

This problem gives you the chance to:
- relate fractions, decimals and percents

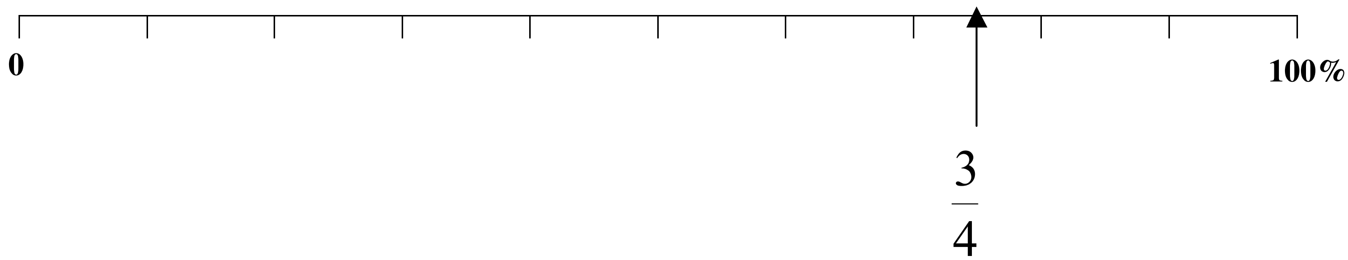
Mrs. Lopez makes sets of cards for her math class.
All the cards in a set have the same value.

	Simple fraction	*Decimal*	*Percent fraction*	*Percent*
Set A	$\frac{3}{4}$	0.75	$\frac{75}{100}$	75%

1. Complete these sets of cards.

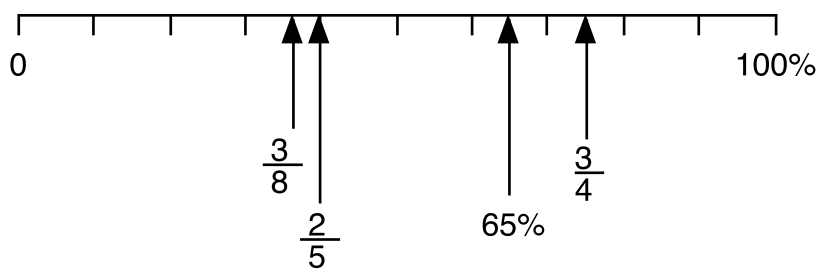
Set B	$\frac{2}{5}$		$\frac{40}{100}$	40%
Set C		0.65		65%
Set D	$\frac{3}{8}$			

2. Show $\frac{2}{5}$, 65% and $\frac{3}{8}$ on the number line below.



7

Percent Cards — Rubric

The core elements of performance required by this task are: • relate fractions, decimals and percents Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: Set B **0.40**	1	
Set C **13/20, 65/100**	2 x 1	
Set D **0.375, 37.5/100 (**accept 375/1000**), 37.5%**	2	
Partial credit One error	(1)	5
2. Values correctly indicated on number line. 	2	
Partial credit One error	(1)	2
Total Points		**7**

Percent Cards

Work the task and look at the rubric. What are the big mathematical ideas being assessed in this task?

What strategies might you expect students to use to find the simple fraction in Set C? Look at student work on C. How many students gave a correct answer of 13/20?__________

Make a list of other answers. How might students have gotten these answers?

Look at student work for set D. How many of your students put:

.375 37.5/100 or 375/1000 37%	.38 38/100 38%	0.37 37/100 37%	0.3 30/100 30%	0.26 26/100 26%	Other

What types of strategies did successful students use?
What misconceptions did you see as you looked at student work?
Did your students show what they were thinking by writing calculations?

What does it mean for something to be "all the same value"?
Do students get opportunities to have discussions about when to round and when rounding is not appropriate?
If the instructions had told students not to round off, what percentage of the students who did round do you think would be able to make sense of the number in the thousands place and make a correct representation for it? What evidence did you consider in their work?

Now look at the work on the number line. How many of your students:

- Could correctly place all the numbers on the number line?
- Added values on the line to show the scale?
- Redid the number conversions?
- How many of your students had values for 3/8 between:

0.375%	Other values between 30-39%	20-29%	70-79%	80-89%	90-100%	Other

- How many of your students had a relative order of:
    - 2/5, 3/8/ 65%?
    - 2/5, 65%, 3/8?

Have your students worked with the number line this year? How can this mathematical representation be used to help students solve problems or check their work? How can this representation be used to show the meaning of operations with rational numbers?

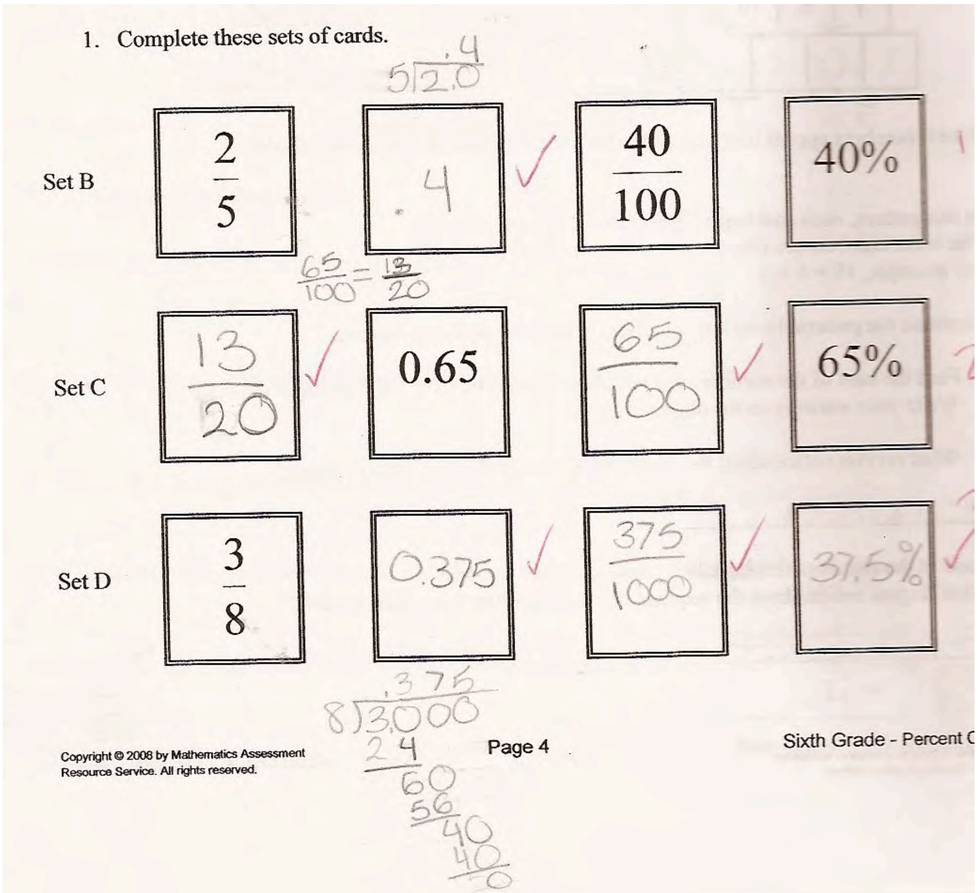
Have you ever used the double number line to help make sense of percents?

What are some other uses of the number line that help students develop a deeper understanding of number?

Looking at Student Work on Percent Cards

Student A uses division to convert the simple fractions to decimals. The student reduces the percent fraction in C to find the simple fraction. The student is able to place all the values on the number line.

Student A



1. Complete these sets of cards.

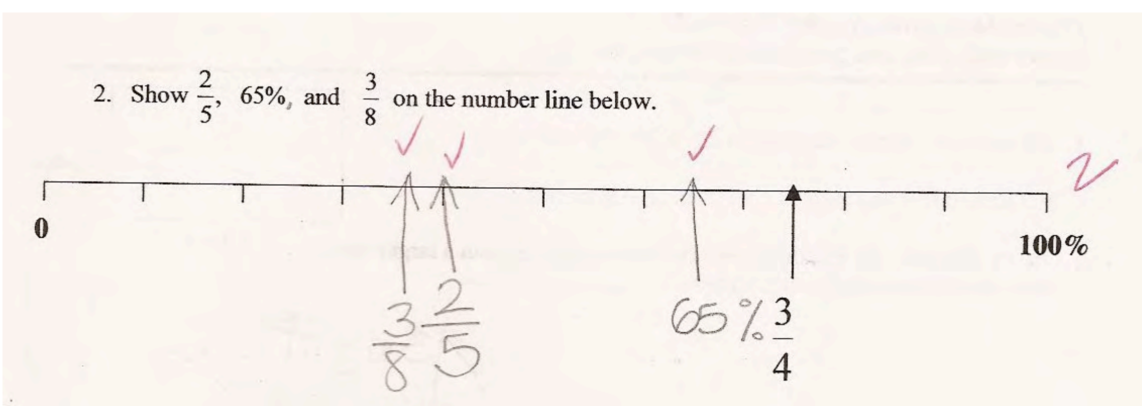
Set B	$\frac{2}{5}$	.4	$\frac{40}{100}$	40%
Set C	$\frac{13}{20}$	0.65	$\frac{65}{100}$	65%
Set D	$\frac{3}{8}$	0.375	$\frac{375}{1000}$	37.5%

$\frac{65}{100} = \frac{13}{20}$

Copyright © 2008 by Mathematics Assessment Resource Service. All rights reserved.

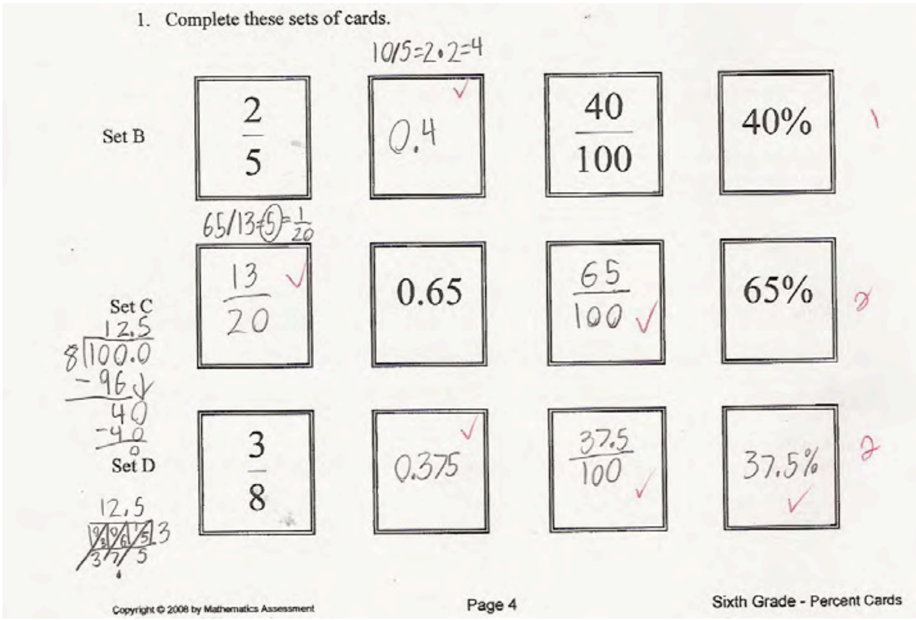
Page 4

Sixth Grade - Percent C

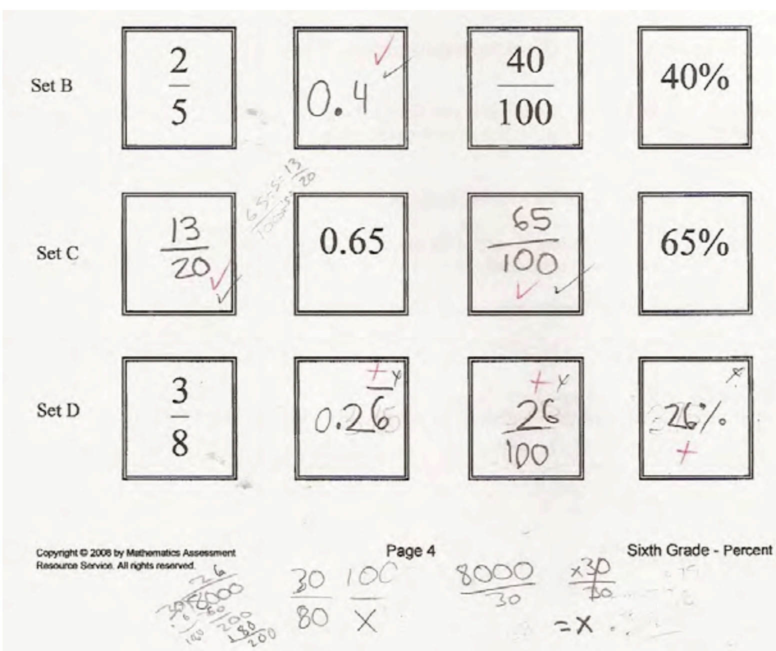


2. Show $\frac{2}{5}$, 65%, and $\frac{3}{8}$ on the number line below.

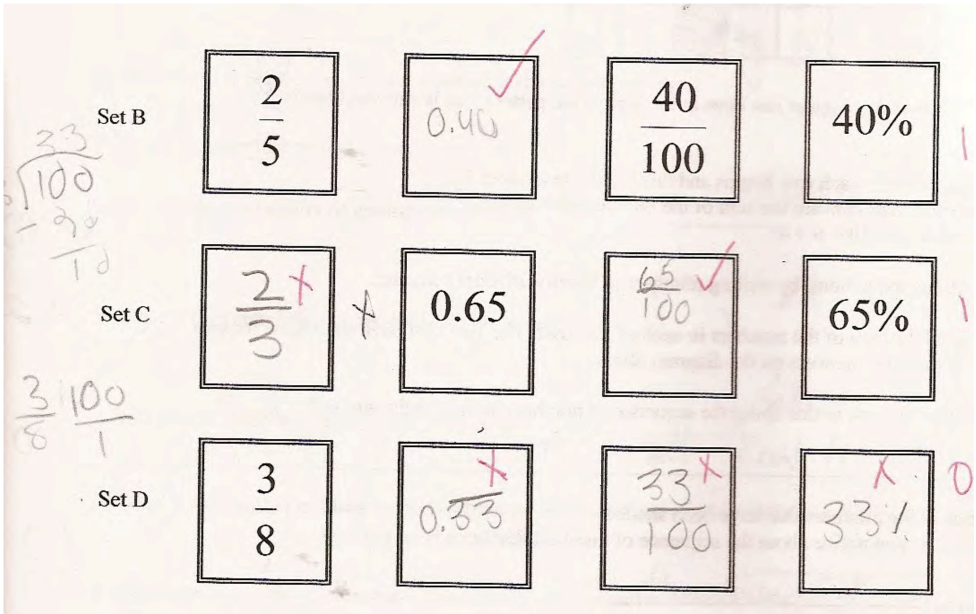
Student B uses a unit fraction approach to thinking about the conversions. If one fifth equals 2/10ths, then 2/5 equals 4/10ths. If 1/20 equals 5/100ths, then 13/20ths equals 65/100ths.



Student C makes a common error by reversing the numbers in the division. The student divides the top number into the bottom number. Notice that the student changes the 3 to a 30 so that the answer will be a decimal. *How do we help students master a procedure but also learn enough understanding of why it works to help them correct errors as they occur? What kind of experiences could push* students thinking about the "whys" of the procedure?

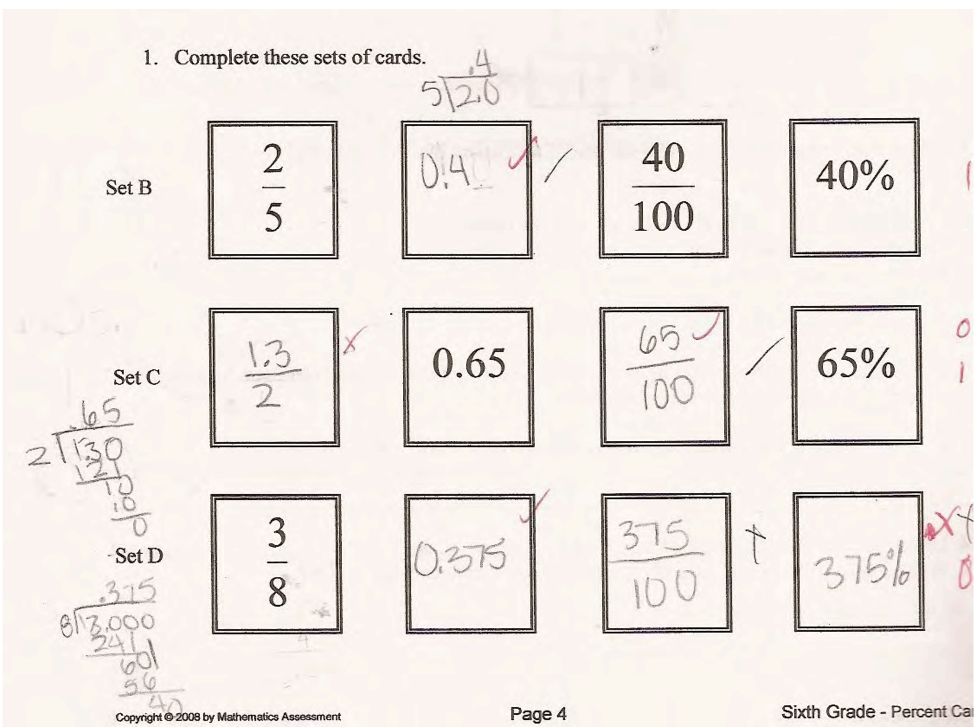


Look at the work of Student D. The student seems to know that .666.. is equal to 2/3. *Can you figure out what the student was doing for the 3/8?*



Set B	2/5	0.40	40/100	40%
Set C	2/3	0.65	65/100	65%
Set D	3/8	0.33	33/100	33%

Student E is able to complete the division for the 3/8, but doesn't know how to work with the decimal to convert to a percent fraction or a percent? *Beyond procedural knowledge, what does a student need to understand about place value and the number system to make these conversions?*



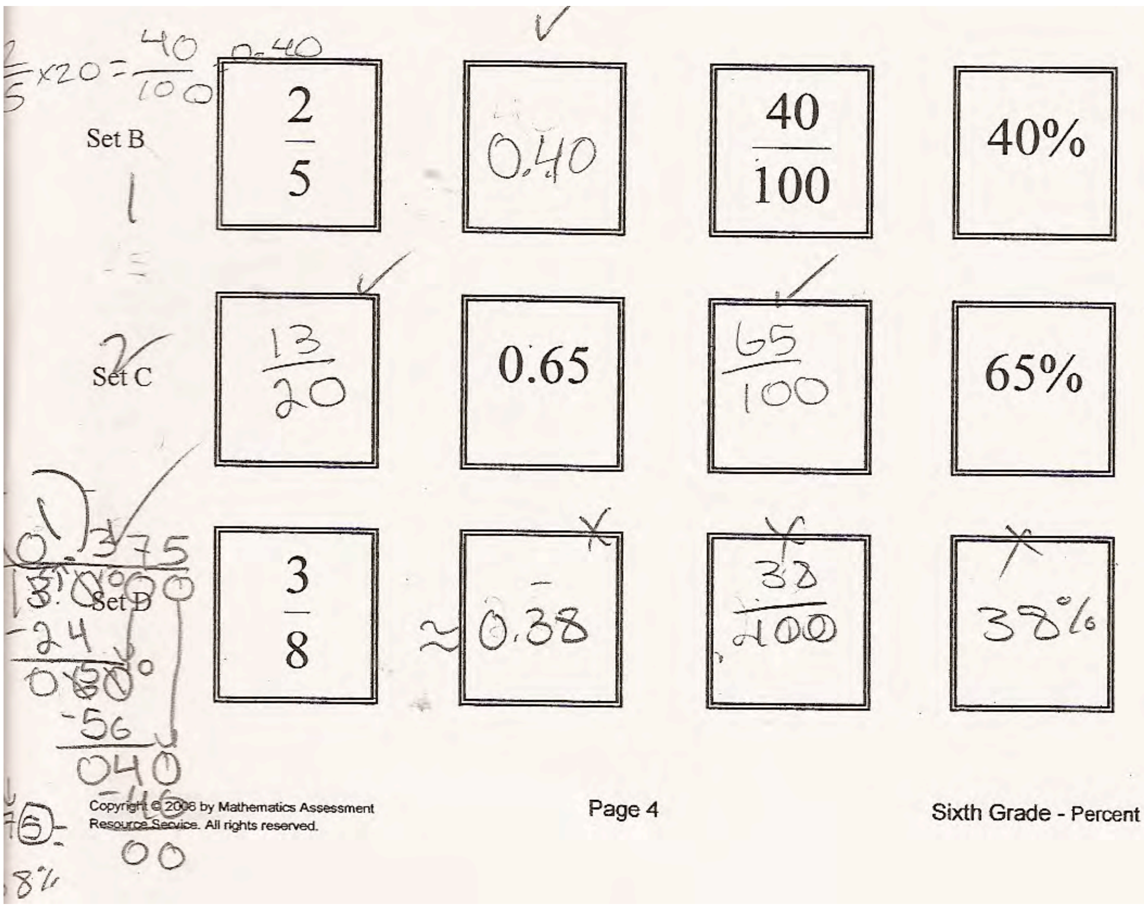
1. Complete these sets of cards.

Set B	2/5	0.40	40/100	40%
Set C	1.3/2	0.65	65/100	65%
Set D	3/8	0.375	375/100	375%

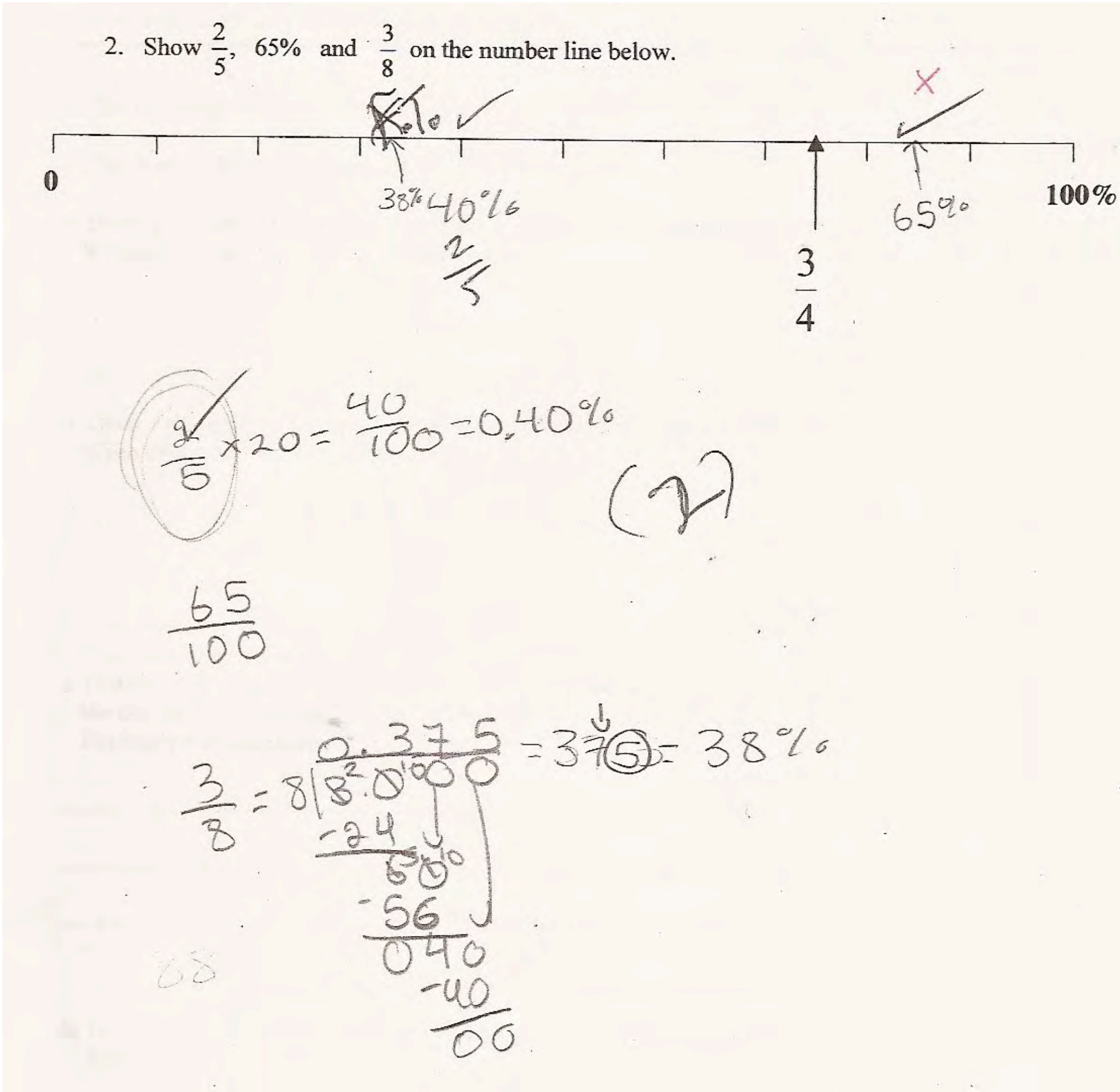
Copyright © 2008 by Mathematics Assessment Page 4 Sixth Grade - Percent Ca

Student F uses two approaches to the conversion of fractions to decimals. In Set B the student finds equivalent fractions. In Set D the student uses division. The student then shows the rounding to go from three to two decimal places. On the second page of the task the student doesn't necessarily connect the values to those on the previous page and reconverts the fractions to decimals and percents. The student can place the fractions correctly on the number line, but misplaces 65%. *What might have confused the student?*

Student F



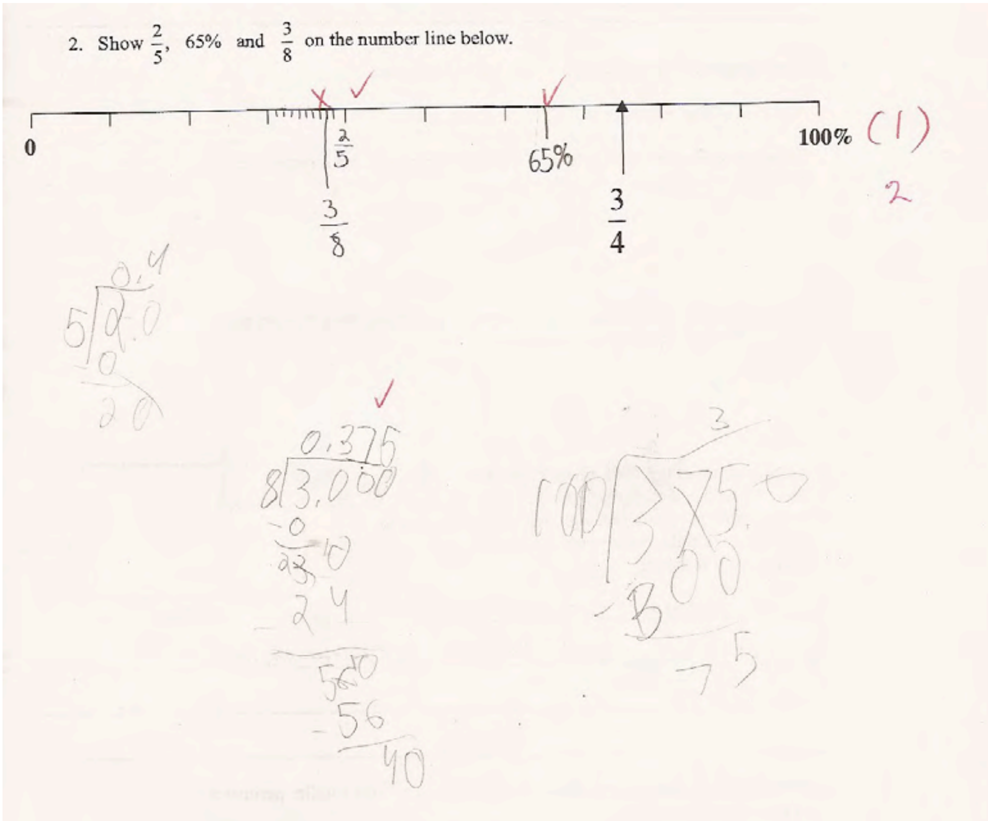
Student F, part 2



2. Show $\frac{2}{5}$, 65% and $\frac{3}{8}$ on the number line below.

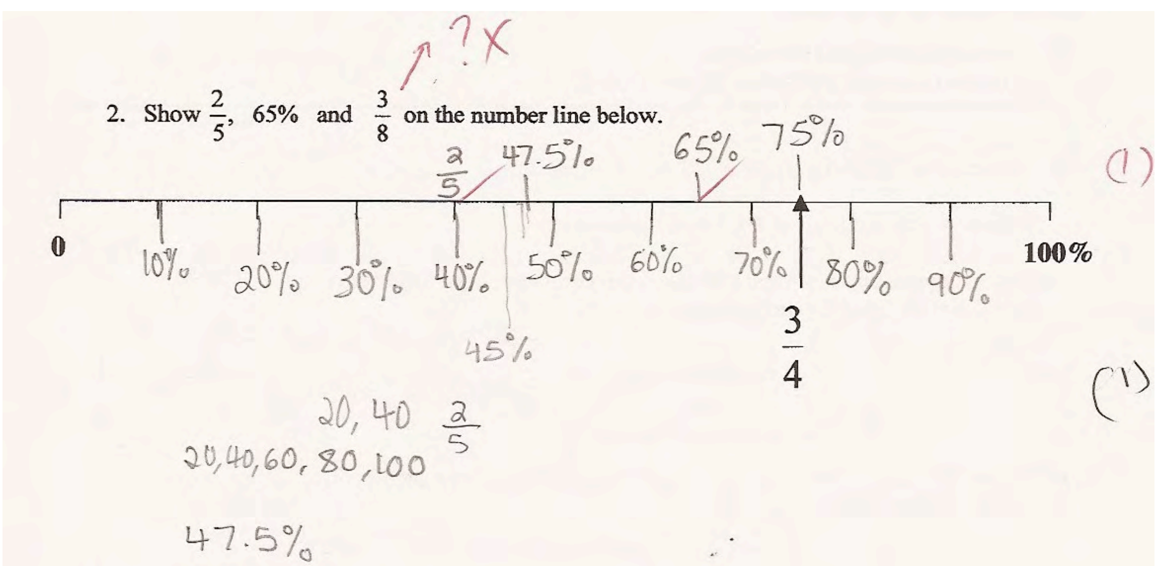
Student G adds marks to the number line to scale the values between 30 and 40%.

Student G



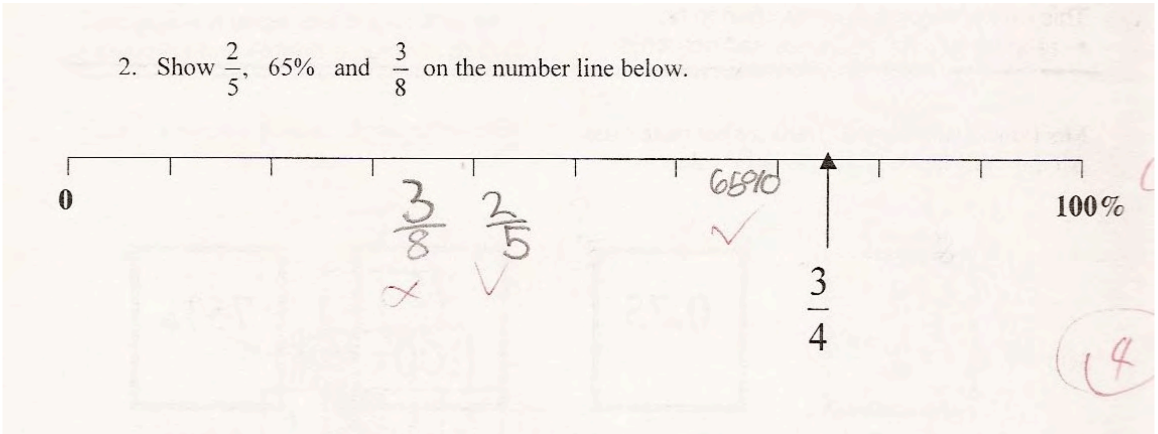
Student H numbers all the lines on the number line to show a scale in percents. The student uses an incorrect value from page 1 of the task for 3/8, but places that value correctly on the line.

Student H



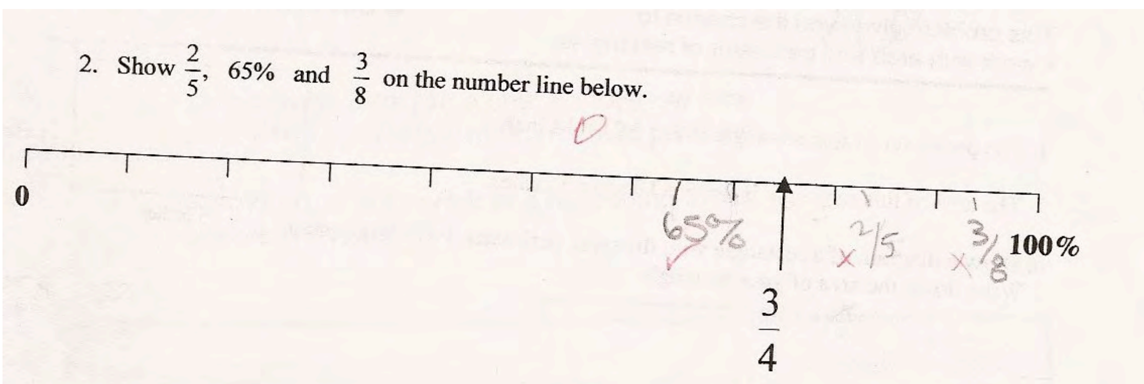
Student I has placed the values in the correct relative order, but does not connect the values to the line.

Student I



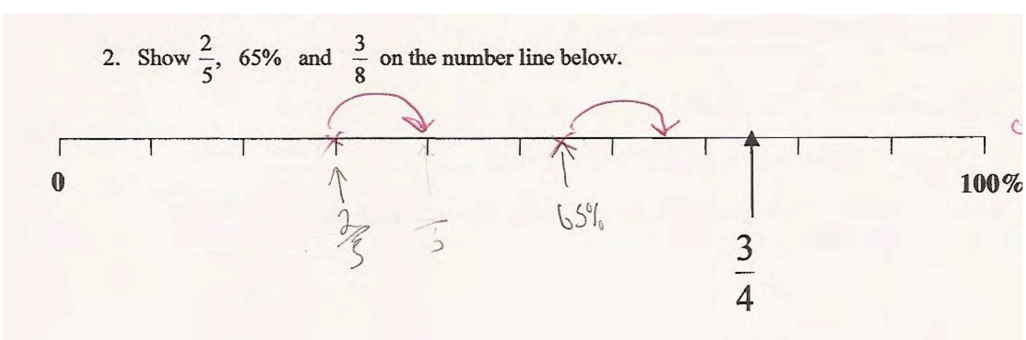
Student J has misplaced the fractions on the number line. The student had correctly identified 2/5=0.40 for Set B and had a value of 3/8 =0.22 for Set B. Notice that neither of these values account for the positions on the number line. *What conjecture can you make for the location of the fractions made by the student?*

Student J



Student K has made the common measurement mistake of counting using the 0 as one. All the locations are off exactly 10%.

Student K



6^{th} Grade Task 4 Percent Cards

Student Task	Relate fractions, decimals and percents. Use number line to locate and compare percents, fractions, and decimals.
Core Idea 1 Number and Operation	**Understand number systems, the meanings of operations, and ways of representing numbers, relationships, and number systems.** • Understand fractions, decimals, and percents as parts of unit wholes and as parts of a collection. • Recognize and generate equivalent forms of commonly used fractions, decimals, and percents. • Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.

The mathematics of this task:
- Understanding decimal place value and working with values in the thousandths place
- Converting between representations using fractions, decimals and percents
- Understanding equivalency
- Using scale to place values on a number line

Based on teacher observations, this is what sixth graders know and are able to do:
- Convert decimals to percents or percents to decimals
- Convert decimals to percent fractions and percents
- Understand the idea of a percent fraction

Areas of difficulty for sixth graders:
- Locating fractions on a number line
- Reducing 65/100 to a simple fraction
- Converting fractions to decimals for values smaller than 100ths
- Understanding equivalency, same value
- Understanding the relationship between percents on the number line and the value of the fractions

Strategies used by successful students:
- Using calculations from part 1
- Marking some other benchmark numbers on the number line, like 1/2, 50%, or 10%

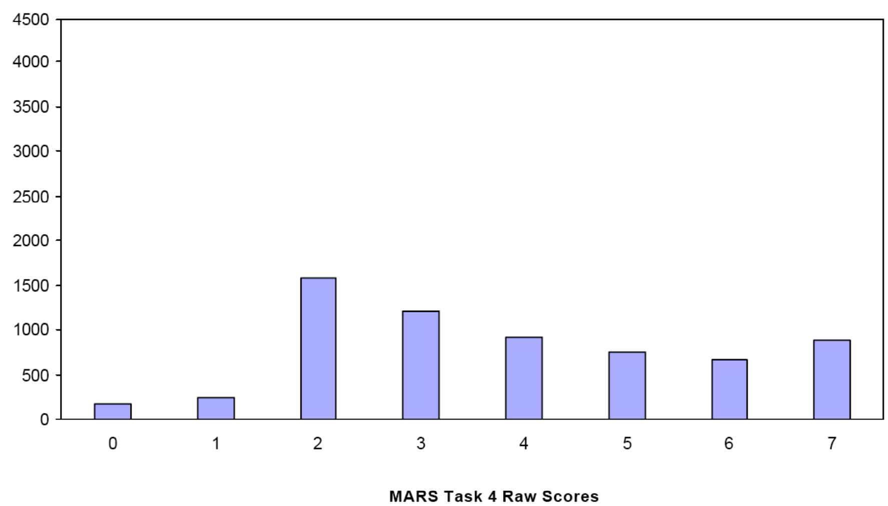
Task 4 - Percent Cards

Mean: 3.84 StdDev: 1.92

Table 33: Frequency Distribution of MARS Test Task 4, Grade 6

Task 4 Scores	Student Count	% at or below	% at or above
0	163	2.6%	100.0%
1	233	6.2%	97.4%
2	1573	31.0%	93.8%
3	1200	49.8%	69.0%
4	909	64.1%	50.2%
5	745	75.8%	35.9%
6	657	86.2%	24.2%
7	879	100.0%	13.8%

Figure 42: Bar Graph of MARS Test Task 4 Raw Scores, Grade 6



The maximum score available on this task is 7 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 94%, could change 2/5 into a decimal and change 0.65 into a percent fraction. Many students, 69%, could also give a simple fraction for 65%. About half the students could find the decimal for 2/5, the fraction and decimal fraction for 65%, and place both these values on the number line. Almost 14% of the students could meet all the demands of the task including changing 3/8 to equivalent decimal and percent values and place 3/8 on the number line. About 3% of the students scored no points on the examine. All the students in the sample with that score did not attempt the task.

Percent Cards

Points	Understandings	Misunderstandings
0		All the students in the sample with this score did not attempt the task.
2	Students could change 2/5 into a decimal and change 0.65 into a percent fraction.	Students had difficulty changing the 65/100 to a simple fraction. About10% of the students put 2/5 between 20 and 30% on the number line. About 7.5% put 2/5 between 40 and 50%. About 7% put 2/5 between 10 and 20%.
4	Students could find the decimal for 2/5, the fraction and decimal fraction for 65%, and place both these values on the number line.	Students struggled with 3/8. Almost 9% did not finish the division or rounded to 0.37%. About 7.5% rounded up to 38%. 5% of the students divided 3 into 8 to get 26%. 5% were able to give the decimal of 0.375, but then used 37/100 for the percent fraction. Students also struggled with how to place 3/8 on the number line. Almost 18% had a relative order of 2/5, 65%, 3/8. 9% had a relative order of 2/5, 3/8, 65%. 13% of the students placed 3/8 between 80 and 90%. 10% of the students placed 3/8 between 20 and 30%.
7	Students could comfortably convert between simple fractions, decimals, percent fractions and percents. Students could work with values in the 1000ths place. Students could place decimals and fractions on a number line scaled in percents.	

Implications for Instruction

Students need practice relating fractions, decimals and percents. Students should be comfortable with fractions that convert to more than 2 decimal places. Students often know procedures for finding equivalent fractions, but don't understand that equivalent means the same size.

Students should be familiar with using a number line to compare numbers. Number lines help students to understand the meaning of adding and subtracting fractions and show how to compare fractions. Models, such as number lines provide students with access to big mathematical ideas, help to justify why the procedures work, and provide students with tools to check their work to see if answers are reasonableness. Models also help students to solve problems and mimic the action of the problems to determine operations. Students at this grade level should be fluent in converting between representations of numbers.

Ideas for Action Research – Understanding Equivalency, Lesson Study

Adults have trouble understanding why concepts are difficult for students. They can't think what is complicated about an idea that they understand. Looking at students expressing their ideas helps to clarify where student thinking breaks down.

At an MSRI (Mathematics Science Research Institute in Berkeley) Deborah Ball interviewed a 6th grade student on equivalent fractions in front of an audience of over 100 math educators from across the U.S. The student seemed confident and willing to talk. The student knew several procedures for working with fractions, but had difficulty explaining why the procedures worked. When probed about placing numbers on the number line, the student reached disequilibrium when 2 different fractions appeared to be located in the same place on the number line. The video can be viewed on line by going to the MSRI website: www.msri.com
Then scroll to Communications and click on: streaming video lectures. On the new screen, scroll to Special Productions and click Critical Issues in Mathematics. Finally scroll down to Assessing Student's Mathematical Learning, March 2004.

This is a good video to view with colleagues. You might discuss the following:

- What does the student understand? Is this procedural knowledge or conceptual knowledge?
- Where does the student's thinking breakdown?
- Why is understanding equivalency so difficult for students? How can we make some of these ideas more explicit when we design lessons on fractions?

After viewing the video, your department might consider developing a series of lessons using number lines to make sense of different topics in your curriculum.

Performance Assessment Task

Percent Cards
Grade 6

The task challenges a student to demonstrate understanding of the relationship between fractions, decimals, and percents. A student must make sense of equivalencies between fractions, decimals, and percents. A student must be able to use the number line to locate and position percents, fractions, and decimals.

Common Core State Standards Math - Content Standards

The Number System

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1), (x–1)(x2+x+1), and (x–1)(x3 +x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
6	2008	7	4	50 %