

Marble Game

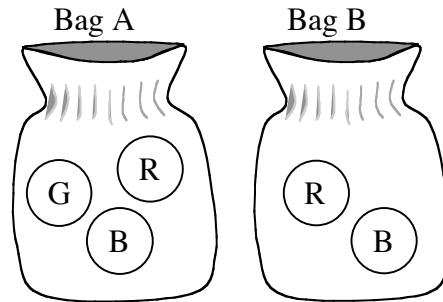
This problem gives you the chance to:

- use probability in an everyday situation

Linda has designed a marble game.

1. Bag A contains 3 marbles – one red, one blue and one green.

Bag B contains 2 marbles – one red and one blue.



To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try?
Show your work.

2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

Explain any differences.

Win (Match)	No Win (No Match)
 /	
	

3. Linda has designed a second game.

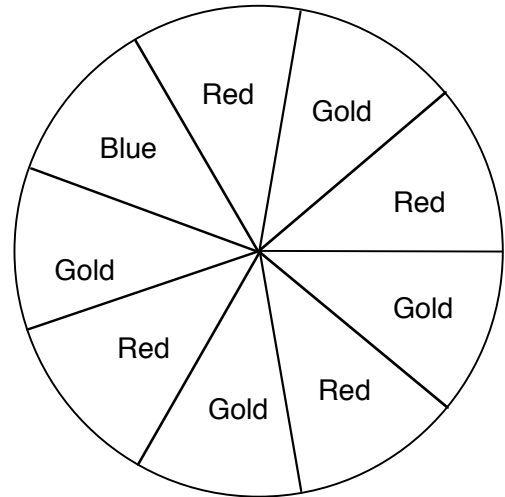
The spinner has nine equal sections.

To play the game, a player spins the spinner.

If the spinner lands on a Gold section,
the player wins a prize.

Does the player have a better chance of winning with
the bag game or the spinner game?

Explain your reasoning.



Marble Game	Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> listed here <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
<p>1. Gives correct answer: $\frac{2}{6} = \frac{1}{3}$</p> <p>Lists all possibilities: RR, RB, BR, BB, GR, GB $p(\text{RR or BB}) = 2/6$ or $1/3$</p> <p>or</p> <p>Shows work such as: Probability $R \cap R = \frac{1}{3} \times \frac{1}{2}$</p> <p style="padding-left: 100px;">Probability $B \cap B = \frac{1}{3} \times \frac{1}{2}$</p> <p>Probability both same color = $\frac{1}{6} + \frac{1}{6}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>or</p> <p>1</p> <p>1</p>	3
<p>2. These results are quite close;</p> <p>but the number of trials is not large enough to give an accurate estimate.</p> <p>or</p> <p>Explains that from these results the experimental probability</p> $= \frac{9}{30} = \frac{3}{10} = 0.3$ <p>The theoretical probability = 0.33 recurring</p>	<p>1</p> <p>1</p> <p>1</p>	2
<p>3. Gives correct answer: the spinner game</p> <p>Shows work such as: the probability of winning on the spinner game is $4/9 = 0.44$ recurring</p>	<p>1</p> <p>1</p>	2
Total Points		7

Marble Game

Work the task and look at the rubric. What are the key mathematical ideas being assessed in this task? _____

What activities or experiences have students had with probability this year to help prepare them for the High School Exit Exam?

How do you help students draw or define sample spaces? What models do they know?

Look at student work on part 1. How many of your students put:

1/3	2/3	4/5	2/5	1/5	Red	Other

Make a list of models students used to figure out probability:

- Organized list
- Formula

Now look at student work of part 2. How many students:

- Knew the probability was $9/30$ or 30% ? _____
- Did not compare to the two probabilities? _____
- Thought that winning was unlikely (nonnumerical)? _____
- Thought there were less wins than losses? _____
- Thought the probability was $9/21$ or $3/7$? _____

What are some of the big ideas about quantifying probability that your students did not understand?

Now look at student work on part 3, finding the probability of the spinner. How many of your students thought

- the spinner was better? _____
- the bag game was better? _____
- they were equal? _____

How many of your students did not attempt to quantify the probability for winning the spinner? _____

Looking at Student Work on Marble Game

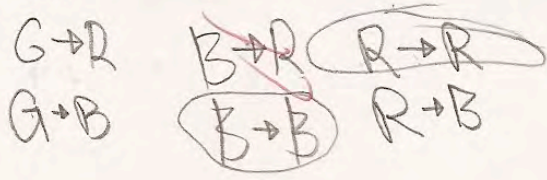
Student A is able to make an organized list to show the sample space in part 1 and quantify the probability. The student does not know how to convert the frequency table into a probability but recognizes that the total is important to the fraction. The student is able to quantify the probability on the spinner and compare it correctly to the bag game.

Student A

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

$$\frac{2}{6}$$

What is the theoretical probability of winning a prize at a single try? Show your work.



2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

$$\frac{1}{30}$$

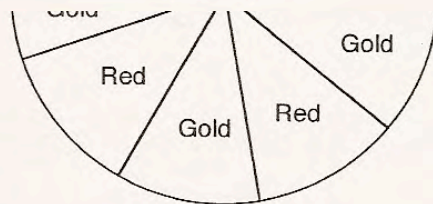
Win (Match)	No Win (No Match)
	0
	0

Explain any differences.

$$\frac{29}{30}$$

Does the player have a better chance of winning with the bag game or the spinner game?

The spinner game



Explain your reasoning.

$$\text{bag game} = .33 \quad \text{spinner game} = .44$$

Student B is able to get the correct probability, but from drawing the sample space incorrectly. *What error has the student made?* The student is able to calculate the experimental probability from the frequency table and understands that the denominators need to be the same to compare the two probabilities. However the student makes the wrong choice when comparing the two. The student has a complete explanation for part 3.

Student B

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try? Show your work.

RR BB (G?)
1 2 3 1 out of 3 ^

$\frac{1}{3}$ ✓ 1

0

0

2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

The win rate is higher than theoretically. ^

Explain any differences.

The win rate was a little higher than the theoretical probability. ^

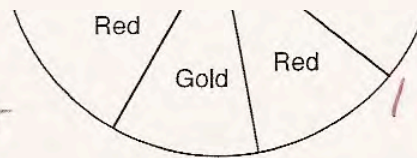
$\frac{10}{30} \cdot \frac{1}{3} = \frac{10}{90}$ but $\frac{9}{30}$ ^

Win (Match)	No Win (No Match)
 9	 21

0 1

Does the player have a better chance of winning with the bag game or the spinner game?

The spinner game has better chances.



Explain your reasoning.

It is because the bag has $\frac{3}{9}$ chance of winning & the spinner is $\frac{4}{9}$. ✓

Student C is able to draw a complete sample space and identify the winning choices, the student even gives the numerical probability. The student does not see the importance of the numerical probability with the idea of likely, unlikely, and certain, etc. dominating the thinking. Notice that in part 2 the student doesn't quantify the experimental probability. The student does give the correct numerical probability in part 3.

Student C

To play this game, a player draws one marble from each bag without looking.
If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try?
Show your work.

Unlikely \times \rightarrow

Possible Outcomes - GB

Gk

RR (matches) = 2 out of 6 chance = 1 out of 3 chance of winning.

RB

BR

Bb (matches)

2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

Unlikely \times

Win (Match)	No Win (No Match)

Explain any differences.

There's not really a difference because the theoretical probability that I solved had a better chance of losing like how it shows in the table. \checkmark

Student D is not very explanatory and relies on numbers. In part 1 the student thinks about a variety of choices, but lands on the correct probability. In part 2 the student quantifies the probability, but makes no attempt to compare the probabilities. In part 3 the student finds the numerical probability.

Student D

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try? Show your work.

$$\frac{2}{6} \text{ or } \frac{1}{3} \checkmark$$

$$\frac{2}{5} \times \frac{1}{5} = 0$$

2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

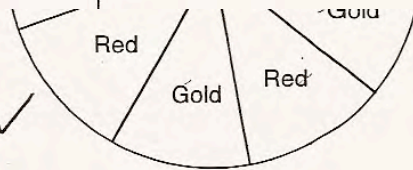
$$\times \quad \frac{9}{30} \checkmark$$

Win (Match)	No Win (No Match)

Explain any differences.

Does the player have a better chance of winning with the bag game or the spinner game?

The Spinner game ✓



Explain your reasoning.

Because $\frac{4}{9}$ ✓ chances is better than $\frac{2}{6}$

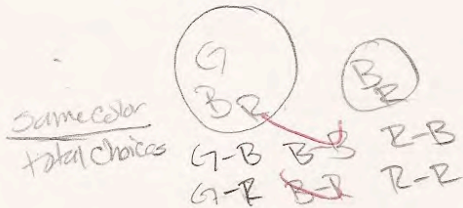
Student E is able to define the sample space and give a definition for finding probability. The student can't turn the definition into a numerical expression. Instead the student quantifies the number of wins. In part 2 the student is also concerned with winning or losing rather than a numerical expression. The student subtracts wins from losses with the frequency table, rather than comparing the probability in 1 with the probability in 2. The student could find the numerical probability for the spinner. *How do we help students to see the relevance for the numerical probability when their focus is on the importance of winning? What would make this important to them?*

Student E

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try? Show your work.

~~2~~ wins



2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

You cant win a lot of times

Explain any differences.

There is a 16 point difference

Win (Match)	No Win (No Match)
	0

0
1
1

Student F gets the correct probability, but for the wrong reasons. The student seems to think about 2 bags, so 50% and 50% to get 100. Then the student divides by the 3 marbles in the first bag. The student does calculate correctly the experimental probability. In part 3 the student incorrectly counts the number of sections on the spinner. The student doesn't combine the winning choices.

Student F

Marble Game

This problem gives you the chance to:

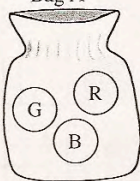
- use probability in an everyday situation

Linda has designed a marble game.

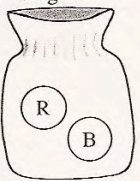
1. Bag A contains 3 marbles – one red, one blue and one green.

Bag B contains 2 marbles – one red and one blue.

Bag A



Bag B



To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try? Show your work.

$0 + 50 + 50 = 100$
 $\frac{100}{3}$

33.33 ✓

2. Here are the results for the first 30 games.

Win (Match)	No Win (No Match)

How do the results in this table and the theoretical probability you found compare?

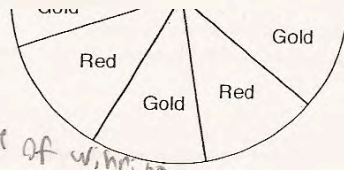
these very close ✓ $\frac{9}{30}$

Explain any differences.

theoretical probability will usually be different from experimental but close.

Does the player have a better chance of winning with the bag game or the spinner game?

✗ bag game $\frac{1}{3}$ - 12.5% chance of winning



Explain your reasoning.

✗ the bag game you have a 33.33% chance to win in the spinner game you have a 12.50% chance to win

Student G is able to make a list for the sample space, but doesn't know how to use it to write a correct probability. The student does not use the total in either part 1 or part 2. The student doesn't understand how to set up the probability for the spinner, and instead seems to compare the reds from the two games.

Student G

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try?
Show your work.

$\frac{2}{3}$

G-B B-B R-B ~~G-R~~ B-R R-R

2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

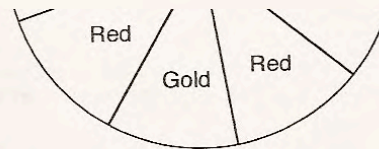
Win (Match)	No Win (No Match)

$\frac{9}{21}$

Explain any differences.

There are losses then win ~~15~~ then you have to make a fraction

Does the player have a better chance of winning with the bag game or the spinner game?



No

Explain your reasoning.

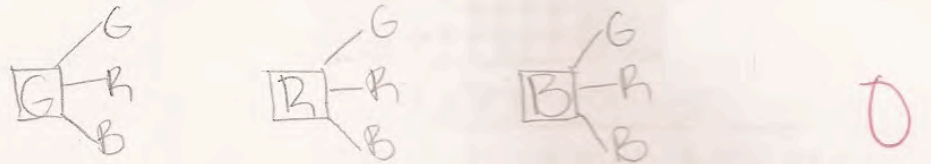
There are also 4 reds to so they would probably lose or win it would just appear.

Student H attempts to draw the sample space, but uses the first bag twice. The student seems to think that games should be fair, have a 50% chance of winning in part 2. The student is able to identify the winning spaces on the spinner, but then compares chances to chances rather than probability.

Student H

To play this game, a player draws one marble from each bag without looking. If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try?
Show your work.



2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

There's no 50% chance

Win (Match)	No Win (No Match)

Explain any differences.

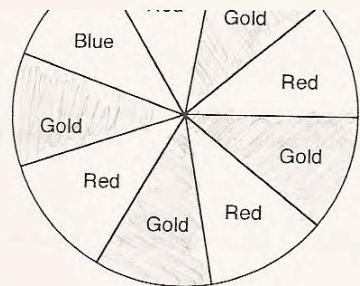
In the marble game there is a lower chance of winning than in the other game

To play the game, a player spins the spinner.

If the spinner lands on a Gold section, the player wins a prize.

Does the player have a better chance of winning with the bag game or the spinner game?

The Spinner game



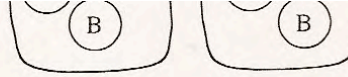
Explain your reasoning.

Because there are just 2 chances on the bag game and 4 on the spinner game.

Student I identifies the winning combinations in part 1, but sets up the probability with total marbles instead of total possibilities. *What experiences help students to understand the distinction between the two?* In part 2 the student seems to compare the winning to not winning rather than set up a probability. In part 3 the student doesn't quantify the fraction for winning on the spinner.

Student I

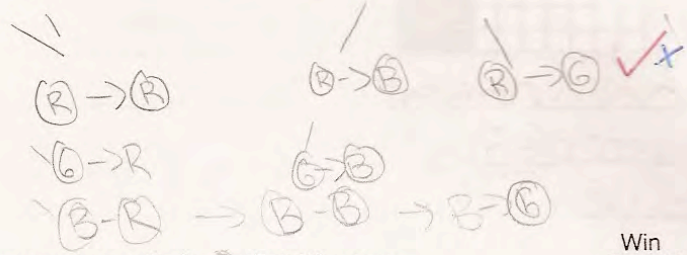
Bag B contains 2 marbles –
one red and one blue. ~~no green~~



To play this game, a player draws one marble from each bag without looking.
 If the two marbles match (are the same color), the player wins a prize.

What is the theoretical probability of winning a prize at a single try?
 Show your work.

$\frac{2}{5}$ ~~trys~~ X C



2. Here are the results for the first 30 games.

How do the results in this table and the theoretical probability you found compare?

$\frac{9}{30}$ $\frac{21}{30} + 9 + 21 = 30/30$

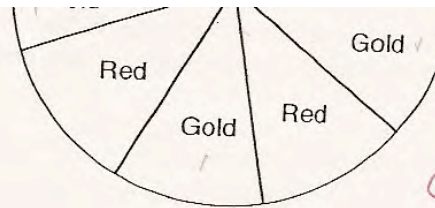
Win (Match)	No Win (No Match)
9	5 5
	5 5
	21

Explain any differences.

you just have to add all of the no together the x 0
lowes + answer you get + the winner wins

Does the player have a better chance of winning with the bag game or the spinner game?

The bag game X



Explain your reasoning.

The bag game has a bigger fraction than the spinner game

Student Task	Use probability in an everyday situation.
Core Idea 2 Probability	<p>Apply and deepen understanding of theoretical and empirical probability.</p> <ul style="list-style-type: none"> • Determine theoretical and experimental probabilities and use these to make predictions about events. • Represent probabilities as ratios, proportions, decimals or percents. • Represent the sample space for a given event in an organized way.

Mathematics of this task:

- Define a sample space for a game
- Identify winning combinations
- Express theoretical probability numerically
- Convert data from a frequency table to an experimental probability
- Compare probabilities by finding commonality (percents, common denominator)
- Find simple probability for a spinner

Based on teacher observation, this is what eighth graders know and are able to do:

- Determine the game with the best probability of winning
- Identify the probability on a spinner

Areas of difficulty for eighth graders:

- Understanding the difference between marbles and the total possibilities in setting up the probabilities
- Defining a sample space for a compound event
- Still thinking about likely/ unlikely or winning/losing instead of in terms of numerical probabilities
- Comparing colors in the bag game and spinner instead of the probabilities
- Using numbers to quantify the probability in the frequency table and spinner

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 8

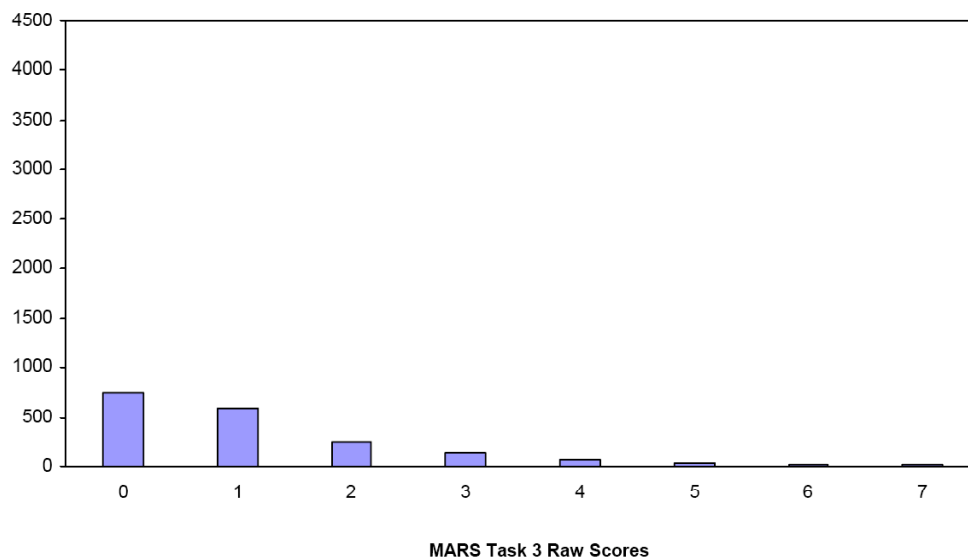
Task 3 - Marble Game

Mean: 1.17 StdDev: 1.40

Table 42: Frequency Distribution of MARS Test Task 3, Grade 8

Task 3 Scores	Student Count	% at or below	% at or above
0	747	40.0%	100.0%
1	592	71.6%	60.0%
2	250	85.0%	28.4%
3	144	92.7%	15.0%
4	75	96.7%	7.3%
5	26	98.1%	3.3%
6	18	99.1%	1.9%
7	17	100.0%	0.9%

Figure 51: Bar Graph of MARS Test Task 3 Raw Scores, Grade 8



The maximum score available for this task is 7 points.

The minimum score for a level 3 response, meeting standards, is 4 points.

More than half the students, 60%, could identify the winning game. Some students, 28%, could also give the numerical probability for winning the spinner game. About 7% of the students could find the probability for winning the bag game (their reasoning may not have been correct). Less than 1% of the students could meet all the demands of the task including defining the sample space for the bag game and giving the theoretical and experimental probability and noticing that they were close in value. 40% of the students scored no points on this task. 92% of the students in the sample with this score attempted the task.

Marble Game

Points	Understandings	Misunderstandings
0	40% of the students in the sample got this score. 92% of them attempted the task.	18% of the students thought the chances of winning the bag game were better. 10% thought the chances were the same for both games. 20% made no attempt to quantify the probability on the spinner game. Some students subtracted winning outcomes (4-2).
1	Students could select the game with the highest probability of winning.	Students could not support their reasoning by quantifying the probability on the spinner.
2	Students could give the probability of winning on the spinner and knew it was a higher probability than the bag game.	Students could not find the probability for winning the bag game. 8% thought the probability was $\frac{4}{5}$. 8% thought the probability was $\frac{2}{3}$. 8% thought the probability was $\frac{2}{5}$.
3	Students could work with the spinner game and find the probability for the bag game.	Students could not give convincing or correct justification for their probability. Many students arrived at a probability of $\frac{1}{3}$ using incorrect logic.
4	Students could find the correct probability for the bag game with correct reasoning, give the correct probability for the spinner game, and compare the two values.	Students could not write the experimental probability or compare it to the probability for the bag game. 13% used the word unlikely instead of giving a numerical value. 13% did not attempt this part of the task. 8% of the students compared winning to losing in the frequency table instead of comparing the frequency table to the theoretical probability in part 1.
7	Students could reason about a compound event, define the sample space, and write a probability. Students could convert a frequency table into an experimental probability. Students could compare probabilities by converting them to decimals, percents or fractions with common denominators. Students could give a probability for a simple event like a spinner.	

Implications for Instruction

Students need more exposure to probability. While in elementary school, students learn about probability in terms of likelihood, in middle school students should start to make progress on quantifying probability. Students need to see the difference between odds (favorable outcomes to unfavorable outcomes) and probability (favorable outcomes to total possible outcomes).

Before any of this can be determined, students need to be able to develop sample space. The spinner is easy because all of the spaces were equal size and could be added together. Students understand generally how to determine probability on a single number cube. They have trouble with compound events: rolling a die and then rolling a second die or spinning a spinner; drawing from two different bags. Students need to practice with strategies like making an organized list or drawing a tree diagram to help them develop all the possible outcomes. Some students still have trouble distinguishing between the actual objects used in the event (marbles in the bag) and the outcome of drawing the red marble.

Some students need to understand that probability is the chance of something happening rather than winning. Many students were only focused on winning the game. Students need to think about the difference between theoretical probability and experimental probability. While many students could quantify the size of the experimental probability, there was little evidence about the difference between chance and expected values.

Ideas for Action Research

With 40% of the students scoring no points on this task, students need to do some serious work with the idea of probability. *While this isn't an eighth grade standard, it is tested on the High School Exit exam.* Probability is also an interesting way to help students review ideas about fractions and decimals, making them feel like they are studying a new topic.

Before trying to formalize the information, students just need to build up some experiential knowledge. They need lots of experiences with games of chance. Some interesting tasks can be found in [Chance Encounters](#) by Education Development Center, Inc.

One group of activities involves describing probabilities as fractions, percents, decimals and ratios. It is called Designing Mystery Spinner Games. See a part of the activity on the next page. A different set of activities is to decide if games are Fair. See examples on following pages. A problem of the month on Fair Games is also available on the SVMII website: www.svmimac.org

Suggested Lesson Plan

1. Problem Solving: *How would you fix these mystery spinner games?*

The process of evaluating and fixing the flawed clues on Reproducible 17 helps students articulate what they have learned about spinners and probabilities. It also helps prepare them for designing their own games. This activity can be done individually or in small groups.

Solutions are provided at the end of the lesson.


#17, Lesson 6

What's Wrong with These Games?

Figure out what's wrong with the games below. Change the clues to fix the problems.

Game 1

- A. You have the highest chance of winning a bike.
- B. The chances of winning a skateboard are $\frac{1}{16}$.
- C. Car has the same chances as skateboard.
- D. Motorcycle has the same chances as roller blades.
- E. You have a 50% chance of winning roller blades.



Game 2

- A. There are four types of snacks on this spinner.
- B. The chances of winning a bag of popcorn are $\frac{1}{12}$.
- C. You will probably get a free apple 10 times out of 30 spins.
- D. You have the same chances of getting carrots as getting popcorn.
- E. You have the smallest chance of getting pretzels.

Reproducible 17 Chance Encounters ©EDC, 1994

Reproducible 17

Reproducible 17

From the Classroom

One student explained a problem in game 1 like this: "Just read D and E. It says motorcycle and Rollerblades are both 50% and there's three more things."

Once students had identified the problems with the clues, I told them "Okay, designers, fix it!" Though students didn't always listen to each other, the groups found ways to work together.

"All of the clues have to equal 100%," said Nick. "There's 93.75 left over."

"How did you figure that?" said Diego.

"You said $\frac{1}{16}$," said Nick, referring to a clue that had just been read.

"Don't change them to percents," said Diego. He then wrote the first clue: Lowest chance of getting bike.

"What about the car?" Nick asked. The boys turned to me and asked if they needed to include all items. I told them they could fix the clues any way they wanted to. "Let's demolish the car!" George declared, and the group agreed to leave out the car.

Their final version of the clues was:

1. Lowest chance of getting bike.
2. Skateboard has the same chance as bike.
3. You'll probably get motorcycle a quarter of the time.
4. 50% chance of getting Rollerblades.

How does this activity press students to practice skills such as using fractions, decimals, and percentages? How does it help students use justification and mathematical reasoning?

▶ **From the Classroom**

We went around the room, each student suggesting a term that could be used in the clues. I recorded students' suggestions on a sheet of chart paper and kept the chart up during the lesson. Students could refer to it as they designed their games, which helped the students write clues.

Reproducible 18

2. Discussion: What kinds of words and numbers can you use to write your own clues?

In preparation for designing their own games, students brainstorm a list of ways to describe chances.

3. Investigation: Design your own mystery spinner game.

Design Your Own Mystery Spinner Games (Reproducible 18) provides guidelines for creating the games. Keeping the spinners to 3–5 parts helps ensure that the spinners can be described well in four clues.

This is a good activity to do in pairs; students often spark each other's creativity and can work together to solve the problems that arise in trying to make the spinner complete.

#18, Lesson 6

Design Your Own Mystery Spinner Game

Spinner Guidelines

- Draw a spinner with 3 – 5 parts.
- Label the spinner parts with the names of foods, music groups, sports, or whatever you want.
- Show the size of each spinner part by labeling it with a fraction or a percent.

Clue Guidelines

- Write at least 4 clues.
- Use a variety of words and numbers to describe the probability of spinning each spinner part.
- Make sure that your clues tell your classmates everything they need to know to draw your spinner.

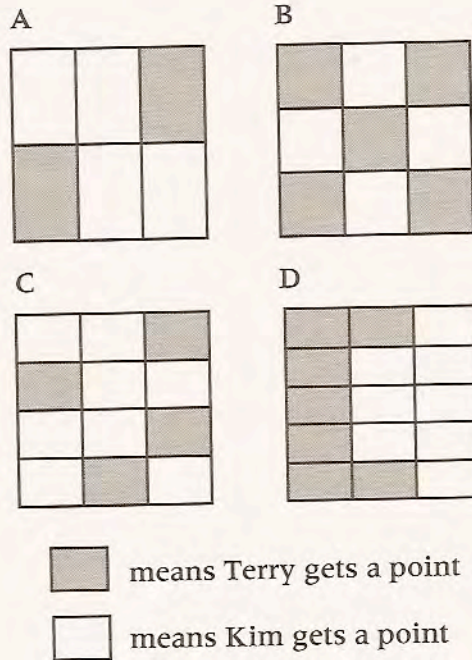
Tip: Write your clues on a different piece of paper from your spinner. Keep the solution hidden from your classmates.

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Reproducible 18

2. Problem Solving: Which grids show the best chances of winning?

In *Rank the Grids* (Reproducible 28), students find two different numerical representations of the player's probability of winning each game. They then rank the probabilities from best to worst chances of winning.



Unlike the 36-box grids students worked with in Step 1, the grids on *Rank the Grids* have different numbers of boxes. Since the grids cannot be compared by counting, students see the value in using fractions, decimals, percents, or ratios to compare and rank the games.

Answers are provided at the end of the lesson.

In this game, students are asked to help Terry decide which is the better game to play. How does this help students practice basic skills? What strategies do you think students might use to rank the games?

4. Investigation: *How can you determine the fairness of games in which players score different numbers of points?*

In *Is This High-Scoring Game Fair?* (Reproducible 29), students use proportional reasoning and outcome grids to explore the relationship between the probabilities and number of points that each player can score. This activity gives students the opportunity to apply and extend their understanding of outcome grids and numeric representations of probabilities.

Name(s) _____ #29, Lesson 9

Is This High Scoring Game Fair?

Rules for the Super Subtraction Game
 Roll two number cubes. Subtract the smaller number from the larger number.
 Player A gets 30 points if the difference is 0 or 5.
 Player B gets 10 points if the difference is 1, 2 or 3.
 Player C gets 45 points if the difference is 4.
 The player with the most points after 18 rolls wins.

- Make an outcome grid on a separate sheet of paper to figure out each player's probability of scoring points in the Super Subtraction Game.
- Complete the table below.

Players	Probability of scoring points on a turn	Number of times each player is expected to score points in 18 turns	Number of points each player can score on a turn	Expected total number of points in 18 turns
A	$\frac{8}{36}$	4	30	120
B			10	
C			45	

- Is the game fair or unfair? Why? If the game is unfair, how would you change the number of points each player can score to make it fair?
- If player A gets 6 points on a turn, how many points would you give each of the other players to make the game fair? Why?
- Come up with your own scoring system to make the game fair.

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Chance Encounters
Reproducible 29

Reproducible 29

In questions 3, 4, and 5 on the reproducible, students change the scoring system for the game to make it fair. By changing the number of points each player can get for winning a turn, students are equalizing the number of points each player is likely to get over the course of several turns. For example, if A is likely to score on 50 out of 100 turns and B on 25 out of 100, then you can equalize the number of points if B can score 2 points on a turn while A can only score 1. The term for describing the combination of the probability of winning a turn and the amount of points scored per turn is *expected value*.

Answers are provided at the end of the lesson.

Performance Assessment Task				
Marble Game				
Grade 8 task aligns in part to CCSSM HS Statistics & Probability				
Task Description				
<p>The task challenges a student to demonstrate an understanding of theoretical and empirical probability. A student must make sense of these probabilities and be able to represent the sample space for a given event in an organized manner. A student must be able to represent probabilities as ratios, proportions, decimals or percents. A student must determine and compare probabilities by finding commonality such as using percents or finding a common denominator. A student must determine the game with the best probability of winning.</p>				
Common Core State Standards Math - Content Standards				
<p>High School – Statistics and Probability - Conditional Probability and the Rules of Probability Understand independence and conditional probability and use them to interpret data. S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</p>				
Common Core State Standards Math – Standards of Mathematical Practice				
<p>MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p> <p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>				
Assessment Results				
<p>This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.</p>				
Grade Level	Year	Total Points	Core Points	% At Standard
8	2009	7	4	8 %