How Old Are They?

This problem gives you the chance to:

- form expressions
- form and solve an equation to solve an age problem

Will is w years old.

Ben is 3 years older.

1. Write an expression, in terms of w, for Ben's age.

____

Jan is twice as old as Ben.

2. Write an expression, in terms of w, for Jan's age.

____

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is ________________ years old

Ben is ________________ years old

Jan is ________________ years old

Show your work.

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4. In how many years will Jan be twice as old as Will? ______________ years

Explain how you figured it out.

____

____

____

____

____

7

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Task 4: How Old Are They?		Rubric
The core elements of performance required by this task are: • form expressions • form and solve an equation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct expression: **w + 3**	1	1
2. Gives a correct expression: **2(w + 3)**	1	1
3 Gives correct answers: Will is **8** years old Ben is **11** and Jan is **22** years old Shows correct work such as: w + w + 3 + 2(w + 3) (allow follow through) 4w + 9 = 41 4w = 32	1 1 1ft.	3
4. Gives a correct answer: in **6** years time **G**ives a correct explanation such as: Will is 14 years younger than Jan so when Will is 14 Jan will be 28. 14 – 8 = 6. Accept guess and check with correct calculations. Solves correct equation.	1 1	2
Total Points		**7**

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How Old Are They?

Work the task. Look at the rubric. How might students use algebra to solve part 3 and part 4? What are the algebraic skills and ideas that students need to work this task?

Look at student work for expressing Jan's age. How many of your students put:

2(w+3)	2w or 2b	Exponents e.g. w^2+3 $(3w)^2$	Parentheses W+3*2 2w+3	Specific value e.g. Ben=6	Used subtraction e.g. 6-w	Other

How are these misunderstanding different from each other? What does each student understand or not understand about variables and how they're used? What are some of the implications for instruction?

Look at the work for the children's ages in part 3. How many of your students:

8,11,22	32,3,6	Other values that total 41	Values that don't total 41	Ages that are not whole numbers	Other

Why do you think students were confused about the solutions? What didn't they understand about the constraints of the problem? How do you set the norms about identifying or defining constraints and showing work to make sure the constraints are met?
How many of your students could use algebra to solve for the ages in part 3?
How many of your students used guess and check?

Look at student work in part 4. How many of your students put:

6	7 or 5	Impossible already twice as old	Negative number	Answer larger than 15	No response	Other

What strategies did successful students use? Tally how many of your students used:

- Make sense of the differences
- Use algebra
- Guess and Check
- Make a table
- Other

Are your students able to apply algebra to a problem solving setting? How can you help them bridge from making a table and guess and check to using some of the mathematical tools of algebra?

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Looking at Student Work on How Old Are They?

Student A is able to set up expressions to represent the ages of the children. Notice that when writing the equation to find the ages of children each expression is defined. The student is able to think about the difference in ages and uses mathematical reasoning to find the time when Jan will be twice as old as will.

Student A

1. Write an expression, in terms of w, for Ben's age.

$w + 3$ ✓ 1

Jan is twice as old as Ben.

2. Write an expression, in terms of w, for Jan's age.

$2(w+3)$ ✓ 1

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 8 years old

Ben is 11 years old

Jan is 22 years old

Show your work.

will, ben, Jan

$w + (w+3) + \boxed{2w+6}$

$4w + 9 = 41$

-9

$4w = \frac{32}{4} = w=8$ $\quad 8+3=11$

$11 \times 2 = 22$

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Student A, part 2

. In how many years will Jan be twice as old as Will? 6 years ✓ 1

Explain how you figured it out.

I saw that their difference in age is 14, so I changed will's age to 14, so 14 x 2 = 28. ✓ 1

Student B is able to represent the situation using algebra to quantify the relationships of the ages and then apply the constraints to set up and solve an equation. Student B is also able to think about growing older as a variable and set up an equation to find out when the Jan will be twice as old as Will. The student is able to use the symbolic language of algebra to represent the problem.

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Student B

How Old Are They?

This problem gives you the chance to:
- form expressions
- form and solve an equation to solve an age problem

Will is *w* years old.

Ben is 3 years older.

1. Write an expression, in terms of *w*, for Ben's age.

$w+3$

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age.

$2(w+3)$

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

$w+w+3+2(w+3)=41$
$2w+3+2w+6=41$
$4w+9=41$
$4w=32$
$w=8$

$8 + 11 + 22 = 41$

Will is 8 years old

Ben is 11 years old

Jan is 22 years old

Show your work.

$22+x=2(8+x)$
$22+x=16+2x$
$-x=-6$
$x=6$

$8+6=14$ $22+6=28$

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Student B, part 2

4. In how many years will Jan be twice as old as Will? in 6 years

Explain how you figured it out.

$22 + x = 2(8+x)$ Jan's age = 2x Will's age

Jan's current age Will's current age x = years

Adding x amount of years to both Jan's and Will's age I then multiplied Will's new age by two to find when Jan will be twice Will's age.

Student C adds a new variable to help express Jan's age in part 2. However the student realizes, when pushed to find the age of all the children, that the new variable won't help solve the problem and makes adjustments to the expression for Jan's age. The student uses a table to find the time when Jan will be twice as old as Will, but does not interpret the elapsed time correctly.

Student C

1. Write an expression, in terms of *w*, for Ben's age. $w+3 = B$

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age. $2b = J$

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

$[(w+3)2] + (w+3) + w = 41$

Will is 8 years old

Ben is 11 years old

Jan is 22 years old

Show your work.

$[(w+3)2] + (w+3) + w = 41$

$(2w+6) + (w+3) + w = 41$
$\quad -3 \quad -3$

$(2w+6) + 2w = 38$
$\quad -6 \quad -6$

$4w = 32$
$/4 \quad /4$

$w = 8$

$8+3 = 11 \cdot 2 = 22$

$8 + 11 + 22 = 41$

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Student B, part 2

4. In how many years will Jan be twice as old as Will? in 6 years

Explain how you figured it out.

$22 + x = 2(8+x)$ Jan's age $= 2 \times$ Will's age

Jan's current age; Will's current age; $x =$ years

Adding x amount of years to both Jan's and Will's age I then multiplied Will's new age by two to find when Jan will be twice Will's age.

Student C adds a new variable to help express Jan's age in part 2. However the student realizes, when pushed to find the age of all the children, that the new variable won't help solve the problem and makes adjustments to the expression for Jan's age. The student uses a table to find the time when Jan will be twice as old as Will, but does not interpret the elapsed time correctly.

Student C

1. Write an expression, in terms of *w*, for Ben's age. $w+3 = B$

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age. $2b = J$

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

$[(w+3)2] + (w+3) + w = 41$

Will is 8 years old

Ben is 11 years old

Jan is 22 years old

Show your work.

$[(w+3)2] + (w+3) + w = 41$

$(2w+6) + (w+3) + w = 41$

$-3 \quad -3$

$(2w+6) + 2w = 38$

$-6 \quad -6$

$4w = 32$

$\frac{4w}{4} = \frac{32}{4}$

$w = 8$

$8+3 = 11 \cdot 2 = 22$

$8 + 11 + 22 = 41$

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Student C, part 2

4. In how many years will Jan be twice as old as Will? 7 years

Explain how you figured it out.

I first subtracted 8 from 22 to get 14. Then I divided 14 by two because each year is the same as two years for Will because you have to multiply his age by 2 to see if it is half as much as Jans. I also made a table to check my work.

	1	2	3	4	5	6	7
W	8	9	10	11	12	13	14
J	22	23	24	25	26	27	28

Student D uses *w* to mean different things. In part 2, *w* is used much the same way as student C as a new variable, without maintaining the referent to Will. Now *w* is used to refer to Ben instead of Will. When pushed to do further thinking, the student is able to correctly describe all the relationships with a single variable. The student struggles with the idea of growing older and only ages Will. Jan does not grow older in this model. *What do you think some of the issues are for students making sense of this idea of elapsed time? Is it an equality problem: whatever you do to one side of the equation you do to the other side? Is it a question of describing the situation first in your own words? What ideas might help this student?*

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Student D

1. Write an expression, in terms of *w*, for Ben's age.

w+3 ✓ ✓ 1 /

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age.

2w x x0

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 8 ✓ years old /

Ben is 11 ✓ years old

Jan is 22 ✓ years old 1

Show your work. ✓

$2(w+3)+w+3+w=41$

$2w+6+w+3+w=41$

$4w+9=41$

$4w=32$

$w=8$

✓

1

4. In how many years will Jan be twice as old as Will? -8 X 0 years 0

Explain how you figured it out.

Jan is already twice as old a Ben. x

She is twice as old and eight years. x0

Twice Ben's age would be 16, and

she is 22.

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Student E is able to use algebra to solve parts 1,2, and 3. However when thinking about the relationship of Jan being twice Will's age, the student misinterprets the meaning of doubling and struggles with the idea of elapsed time for both children. *How do we help students to develop the habit of mind of checking their work for sense-making? Is it reasonable to have a negative amount of time in this situation? Do students in your class have enough opportunities to make sense of answers in context?*

Student E

4. In how many years will Jan be twice as old as Will? -6 XX years 0

Explain how you figured it out.

8×2=16 Jan is 22 years old. 22 is greater than 16 so I subtracted 22 from 16 & got 6 years. X 0.
Since she's already older than 2 times, I added the negative. X

Student F is confusing variable with a "fixed" solution. In part one Ben is given an exact age. In part 2 the student is trying to find a number sentence for twice Ben's age or 6, but does compute correctly with the negative number. In part 3 the student ignores all the previous work and the relationships between the students' ages and just finds a numerical solution to the total ages equals 41. The student either doesn't understand or ignores the prompt in part 4. *How do we help develop the big idea of variable early in the curriculum? How do help students distinguish between an unknown and a variable? What would be your next steps with this student?*

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Student F

1. Write an expression, in terms of *w*, for Ben's age. $w=3$ x x 0 0

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age. x $w=-9+3$ x 0 0

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 20 x years old x 0 0

Ben is 20 x years old x 0 0

Jan is 1 years old x 0

Show your work.

$20+20+1=41$

x

0

4. In how many years will Jan be twice as old as Will? 19 x x years 0 0

Explain how you figured it out.

Well I knew that 20 +20 = 40 so x 0
than I though if I add 1 1 t x 0
will make 41 so that's
how I got 41 as the answer.

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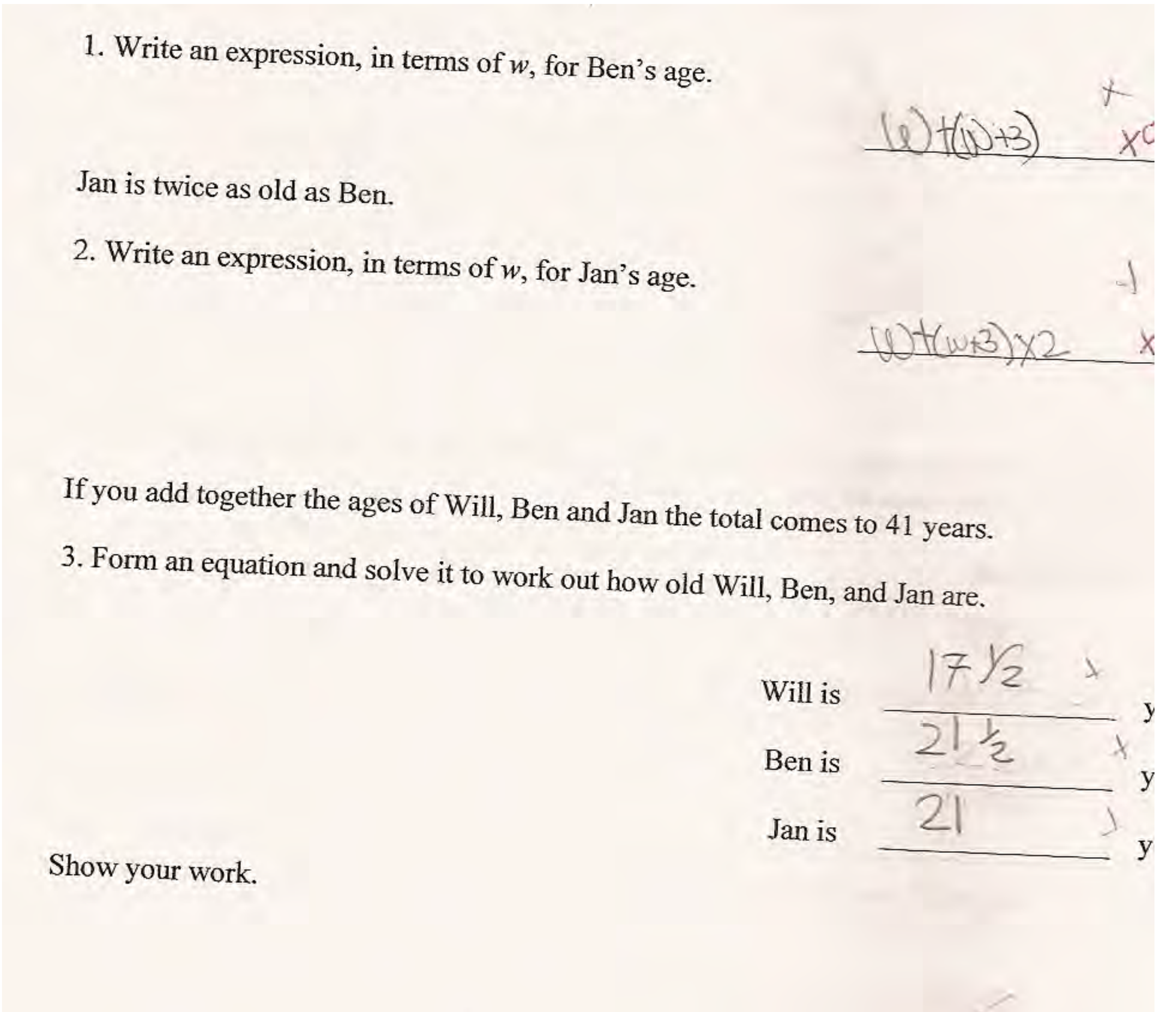
Student G seems to understand the relationships of the students ages in terms of the variable *w*, but then seems uncomfortable with a simple expression and adds on Will's age to each part. The student doesn't use the relationships to find the ages in part 3 and the student doesn't use the constraint of having the ages add to 41. The student leaves part 4 blank. The thinking of Student H may show how the student found the original
17 1/2.

Student G



1. Write an expression, in terms of w, for Ben's age.

(w)+(w+3)

Jan is twice as old as Ben.

2. Write an expression, in terms of w, for Jan's age.

(w+(w+3))x2

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 17 1/2 y

Ben is 21 1/2 y

Jan is 21 y

Show your work.

Student H confuses doubling with using an exponent to square a number. However when using numbers instead of symbols, the student knows to multiply by 2. *Do we give students adequate opportunities to relate symbolic notations to numerical expressions? What is involved in developing fluency with mathematical notation and how can we foster that in our classrooms?* The student tries to work backwards from the doubling by doing a series of "dividing by two". Notice that the totals add to 31 instead of 41. Student H also gives the most popular incorrect answer for part 4. *How do we help students develop a sense of "tinkering with an idea", before making a conclusion?*

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Student H

1. Write an expression, in terms of w, for Ben's age.

w+3=Ben's Age

Jan is twice as old as Ben.

2. Write an expression, in terms of w, for Jan's age.

w^2+3=Jan's Age

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

(Will's Age)= 5½

5½ + 3 =(Ben's Age) 8½

8½ × 2 = 17(Jan's Age)

Will is 5½ years old

Ben is 8 almost 9 years old

Jan is 17½ years old

Show your work.

2)41.0 = 20.5

20.5 − 3.0 = 17.5

2)17.50 = 8.75

8.75 − 3.00 = 5.75

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4. In how many years will Jan be twice as old as Will? None years

Explain how you figured it out.

Jan is already over twice Wills age

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Student I uses an incorrect operation to define Ben's age. The student uses values that match the relationships described in part 1 and 2, but ignores the constraint that the ages should add to 41. In part 4 the student feels that all the numbers in part 3 were doubles, but ignores that constraint that it is Jan who should be twice as old as Will and doesn't take into account the idea of both students growing older.

Student I

1. Write an expression, in terms of *w*, for Ben's age.

w-3 ✓ 0 x

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age.

12w-3 x 0 x

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 3 x years

Ben is 6 x years

Jan is 12 years

Show your work.

3x2=6

6x2=12 x

4. In how many years will Jan be twice as old as Will? 3 x 0 x years

Explain how you figured it out.

I figured this out because I doubled the numbers. x x

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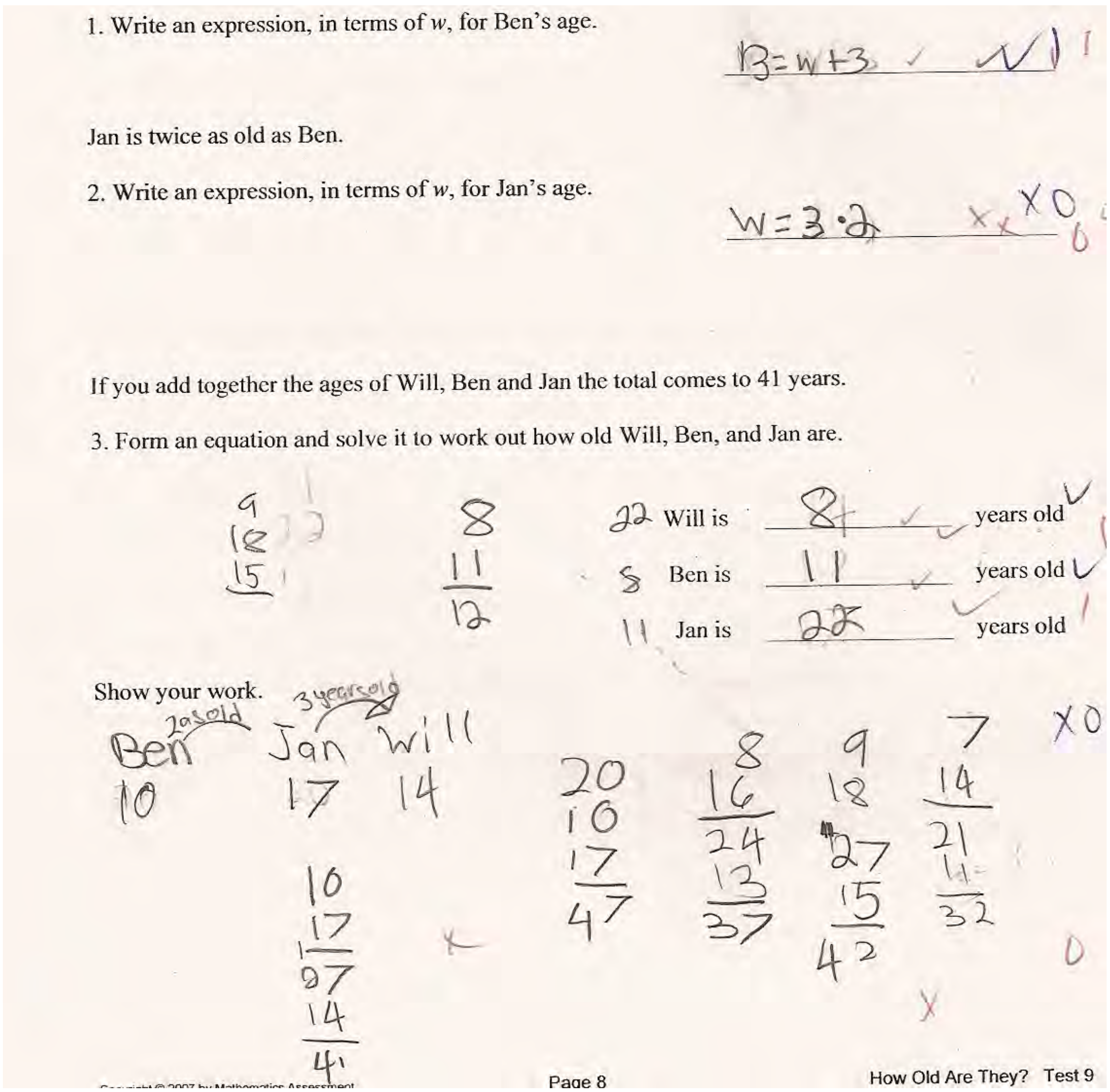
Student J is unable to write an expression for Jan's age, even though the guess and check work shows that the student understands the mathematical relationships. Again, there is a disconnect between the mathematical ideas and how to express it symbolically. The student has the nice habit of mind to be persistent in trying to find the time when Will will be half of Jan's age, however the student seems to only consider Will aging in groups of eight years. *What do you think is causing the confusion?*

Student J



1. Write an expression, in terms of *w*, for Ben's age.

B=w+3

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age.

W=3·2

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is 8 years old

Ben is 11 years old

Jan is 22 years old

Show your work.

Ben 2 as old Jan 3 years old Will

10 17 14

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Student J, part 2

8	22
8	22
16	44
8	22
24	66
8	22
32	88
40	110
8	122
48	132
8	22
56	154
8	22
64	176
8	22
72	198
80	220
8	22
88	242

88	242
8	22
96	264
8	22
104	286
8	22
112	308
8	22
120	330
8	22
128	352
8	22
136	374
8	22
144	396
	22

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MARS Test Task 4 Frequency Distribution and Bar Graph, Course 1

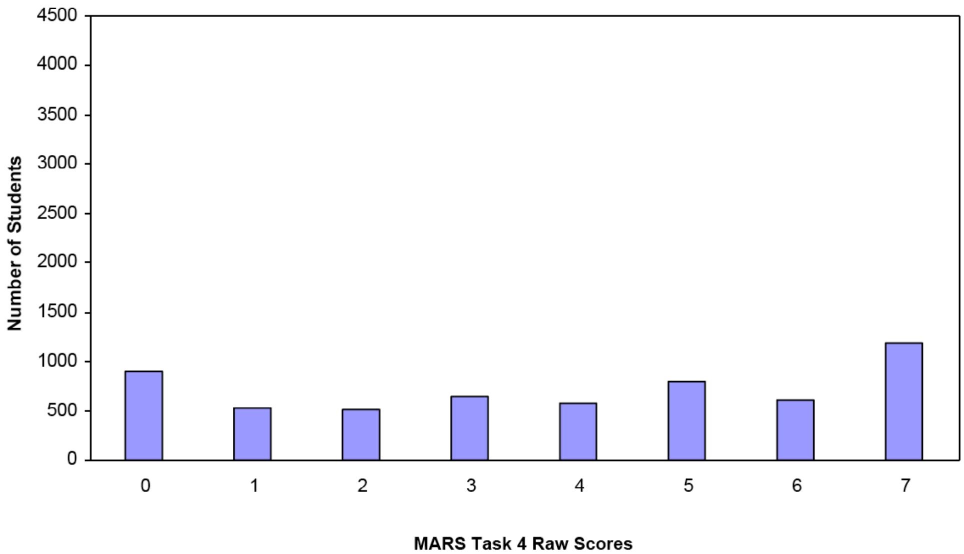
Task 4 - How Old are They?

Mean: 3.79 StdDev: 2.48

Table 48: Frequency Distribution of MARS Test Task 4, Course 1

Task 4 Scores	Student Count	% at or below	% at or above
0	898	15.6%	100.0%
1	520	24.7%	84.4%
2	502	33.4%	75.3%
3	649	44.7%	66.6%
4	572	54.7%	55.3%
5	803	68.7%	45.3%
6	610	79.3%	31.3%
7	1190	100.0%	20.7%

Figure 57: Bar Graph of MARS Test Task 4 Raw Scores, Course 1



The maximum score available for this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 84%, were able to write an algebraic expression for an additive relationship. More than half the students, 66%, could write an additive expression and nd the ages for the 3 students which met all the constraints of the problem. Almost half the students, 45%, could write an expression for an additive relationship, nd the ages of the 3 students, and nd a strategy to correctly calculate the time when Jan will be twice as old as Will. Almost 21% of the students could meet all the demands of the task including writing a multiplicative relationship involving distributive property and writing and solving an equation to nd the ages of the 3 children. Almost 16% of the students scored no points on this task. 69% of the students with this score attempted the task.

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How Old Are They?

Points	Understandings	Misunderstandings
0	69% of the students with this score attempted the problem.	Students were confused about writing an expression for Ben's age. Some tired to give a numerical value, such as w=3. Others used an incorrect operation, such as y=w-3 or 3w.
1	Students could express symbolically an additive relationship.	Students didn't understand the constraints of the relationships. Almost 5% of the students thought Will was 32. 10% of the students gave answers where Jan was not twice Ben's age. More than 20% gave answers that did not add to 41.
3	Students could write an additive expression and find the ages of the three children.	Students did not use algebra to find the ages of the students. More than 30% of the students used guess and check.
5	Students could write an additive expression, find the ages of the three children, and find the elapsed time for when Jan would be twice as old as Will.	17% of the students did not attempt part 4 of the task. 10% thought it was impossible because Jan was already more than twice Will's age. 4% made tables but couldn't interpret the elapsed time and thought it would be 7 years. About 3% gave negative answers for elapsed time.
6		Students with this score struggled with using algebra to solve for the students ages in part 3 or writing an algebraic expression for Jan's age. 18% 11% added a new variable, 2b. 8% wrote 2w, ignoring the "+3". 5% forgot the parentheses, e.g. w+3*2 or 2w+3. 3% tried to use exponents.
7	Students could express relationships using symbolic notation, set up and solve an equation for the children's' ages, and find out when Jan's age would be twice Will's.	

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Implications For Instruction

Students at this level need more opportunities to use algebra in practical situations or apply algebra to word problems. Many of the subtleties of meaning do not arise with the structure provided by context. One question that arises from using context is the idea of the number of variables needed to describe a situation; learning the importance of using one variable to quantify the relationships between different parts of the problem. (A good example of this is provided in the Middle School Mathematics Teaching Cases by Jo Boaler and Cathy Humphreys, which includes a lesson plan, questions for reflection, and video of the lesson.) Students at this grade level should be able to quantify simple relationships using one variable. Students should know when to use parenthesis within an expression. Students should be able to combine expressions to form an equation and solve a simple equation with one unknown. Students should be comfortable with mathematical notations, e.g. understanding the difference between multiplying by 2 and squaring a number or how to use parentheses to define order of operations or a whole quantity being doubled. Writing equations in context also brings up important ideas about equality. To make an equation, the student needs to think about what quantities can be made equal to each other. What do I have to do to one part to make it equal to some other part? How will this help find what I am looking for? How does it relate to the action of the problem?

Ideas for Action Research – Re-engagement

One useful strategy when student work does meet your expectations is to use student work to promote deeper thinking about the mathematical issues in the task. In planning for re-engagement it is important to think about what is the story of the task, what are the common errors and what are the mathematical ideas I want students to think about more deeply. Then look through student work to pick key pieces of student work to use to pose questions for class discussion. Often students will need to have time to rework part of the task or engage in a pair/share discussion before they are ready to discuss the issue with the whole class. This reworking of the mathematics with a new eye or new perspective is the key to this strategy.

In this task, there are two issues that might be interesting to explore or re-engage in. The first issue is the idea of using a variable. Try taking two or three interesting pieces of student and work and using it to set up cognitive dissonance or disequilibrium. For example you might pose the following question:

Sally is confused about how to write the expressions. When she talks to her partner, Frieda, Frieda tells her that Ben is equal to w + 3 and Jan is equal to 2w. However her tablemate, Jaime, agrees that Ben is w+3, but says that you need a new variable to find Jan's age, 2b. As if this was not confusing enough, her third tablemate said that Ben is w +3 and Jan is w^2 +3. See if you can help Sally sort through this information. Who do you think is right or are they all wrong? Give reasons to support your thinking.

How do these questions push students to rethink or re-engage with the mathematics of the task? How do the questions foster discussion about the big ideas of using variables? How does the struggle to justify and convince help students to clarify their own thinking or look at the mathematics from a different perspective?

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The second issue is the idea of aging. Again look at student work, to help pick examples of students thinking to help pose the question. For example:

I overheard a table in a different class discussing part 4, in how many years will Jan be twice as old as Will. Lily thinks that Jan is already twice as old as will. Eugene thinks in 3 years Will will be twice as old as Jan, because 8+ 3= 11 and 11 *x* 2 = 22. Cody says I think you are both wrong. They both need to get older. But I don't know how to write that. What advice could you offer the group? Who do you think is right? How can you convince the others that they have made a mistake?

These types of discussions give students the opportunity to hone in their logical reasoning skills, clarify mathematical ideas, and practice using academic language. What questions would you pose? Why?

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Performance Assessment Task
How Old Are They? Grade 9
This task challenges a student to use knowledge of algebraic notation to write expressions, including how to quantify the distributive property, and combine the expressions into equations for a context situation about ages. A student must be able to solve three equations with two variables. A student must be able to evaluate the meaning, utility, and reasonableness of results of symbolic manipulations. A student must be able to use the knowledge of equality to change an equation based on elapsed time.
Common Core State Standards Math - Content Standards
<u>**High School – Functions – Building Functions**</u> **Build a function that models a relationship between two quantities.** F-BF.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *b.* Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* **Build new functions from existing functions.** F-BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)(x-1)$ for $x \neq 1$.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions.

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They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2007	7	5	45%

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