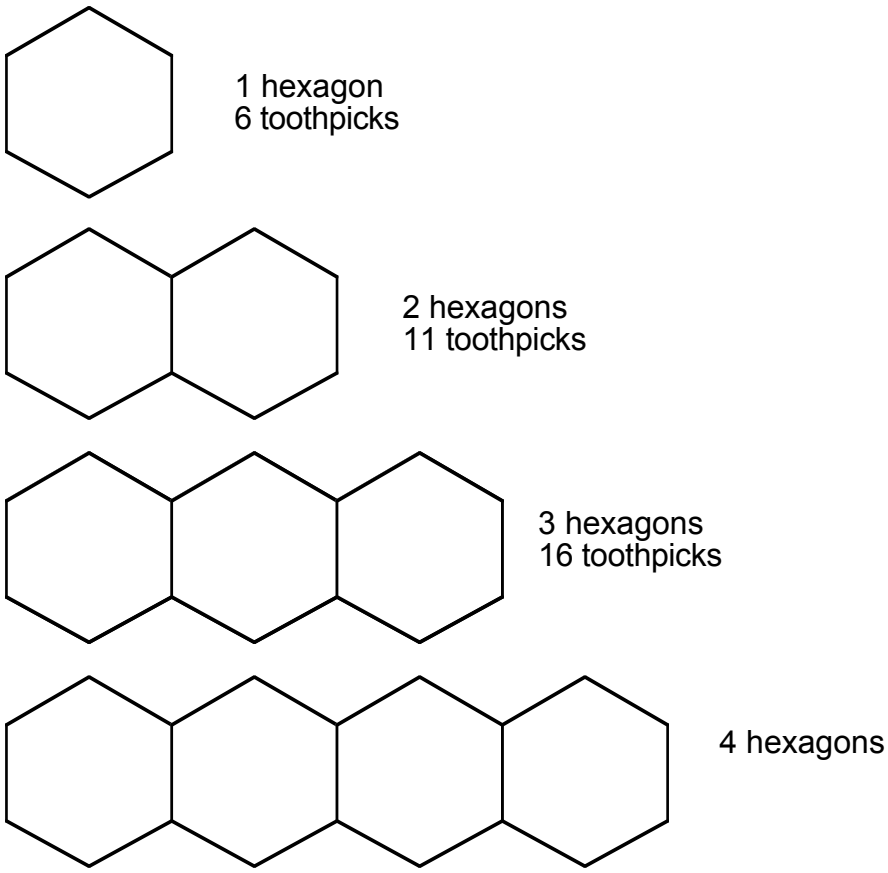
Hexagons in a Row

This problem gives you the chance to:
- find a pattern in a sequence of diagrams
- use the pattern to make a prediction

Joe uses toothpicks to make hexagons in a row.



Joe begins to make a table to show his results.

Number of hexagons in a row	1	2	3	4
Number of toothpicks	6	11		

1. Fill in the empty spaces in Joe's table of results.

2. How many toothpicks does Joe need to make 5 hexagons? _____

   Explain how you figured it out.

   _____

   _____

3. How many toothpicks does Joe need to make 12 hexagons? _____

   Explain how you figured it out.

   _____

   _____

   _____

4. Joe has 76 toothpicks.
   How many hexagons in a row can he make? _____
   Explain how you figured it out.

   _____

   _____

   _____

Hexagons in a Row	Rubric	
The core elements of performance required by this task are: • find a pattern in a sequence of diagrams • use the pattern to make a prediction Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **16** and **21**	1	1
2. Gives correct answer: **26** Gives correct explanation such as: I added on 5: accept diagrams	1 1	2
3. Gives correct answer: **61** Gives correct explanation such as: The first hexagon needs 6 toothpicks; each extra needs 5. $6 + 11 \times 5 =$ Accept diagrams or adding on.	1 1	2
4. Gives correct answer: **15** Gives correct explanation such as: The first hexagon needs 6 toothpicks; each extra needs 5. $76 - 1 = 75, \quad 75 \div 5 = 15$ Accept diagrams	1 1 1	3
Total Points		**8**

5th Grade – Task 2: Hexagons in a Row

Work the task. Look at the rubric.

What do you think are the key mathematics the task is trying to assess?

Look at student work for part 3. How many of your students put:

61	60	66	63	62	56	Other

What kind of strategies did your students use?

21x3= 63	Continue the table	Multiply by 5 + 1	Repeated addition	Draw & Count	Multiply by 5 + 6	Multiply by 6 – shared sides	7x5 + 21

- Which of these strategies works? Which doesn't? Can you explain using the diagram why it works or what needs to be changed to make it work?
- Does this exercise make you think about the big ideas of the task differently?

Now look at student work for part 4. How many of your students put:

15	14	15r1	13	More than 20	Other

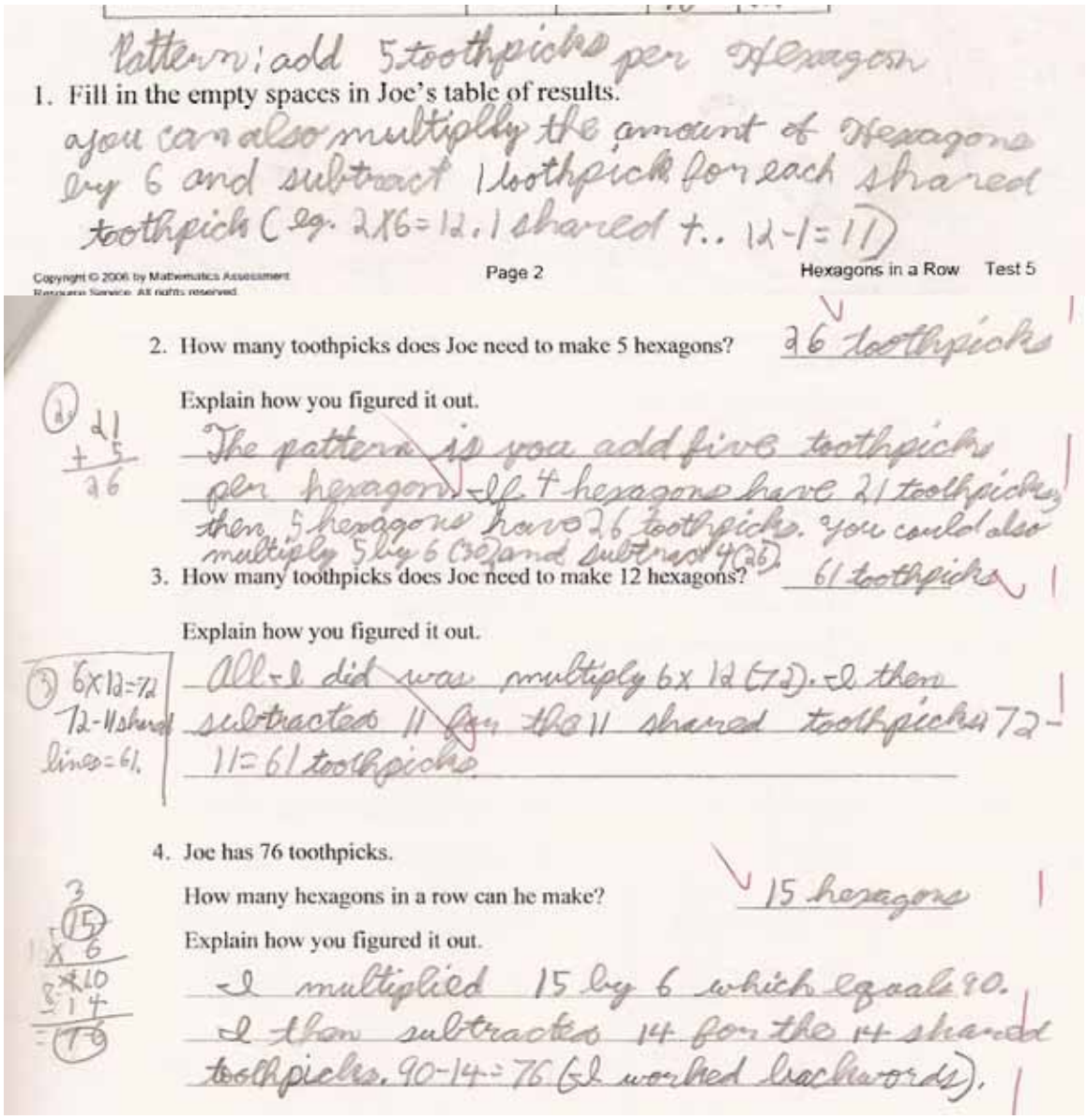
Besides continuing the table and drawing and counting, what strategies helped students to get the correct answer?

What did they have to think about in terms of the structure of the pattern to work backwards?

Looking at student work on Hexagons in a Row

Student A notices that while all hexagons have 6, when then join together one side overlaps. The student is able to quantify the overlaps by subtracting out the number of hexagons minus one. This generalization will be expressed algebraically, at later grades, as t= 6x – (x-1); where t= number of toothpicks and x = number of hexagons, (x-1) represents the number of overlaps for any part of the sequence.

Student A



Pattern: add 5 toothpicks per Hexagon

1. Fill in the empty spaces in Joe's table of results.

you can also multiply the amount of Hexagons by 6 and subtract 1 toothpick for each shared toothpick (eg. 2X6=12. 1 shared t.. 12-1=11)

2. How many toothpicks does Joe need to make 5 hexagons? 26 toothpicks

Explain how you figured it out.

21 + 5 = 26

The pattern is you add five toothpicks per hexagon. If 4 hexagons have 21 toothpicks, then 5 hexagons have 26 toothpicks. You could also multiply 5 by 6 (30) and subtract 4 (26).

3. How many toothpicks does Joe need to make 12 hexagons? 61 toothpicks

Explain how you figured it out.

6X12=72 72-11 shared lines=61.

All I did was multiply 6 x 12 (72). I then subtracted 11 for the 11 shared toothpicks 72-11=61 toothpicks.

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15 hexagons

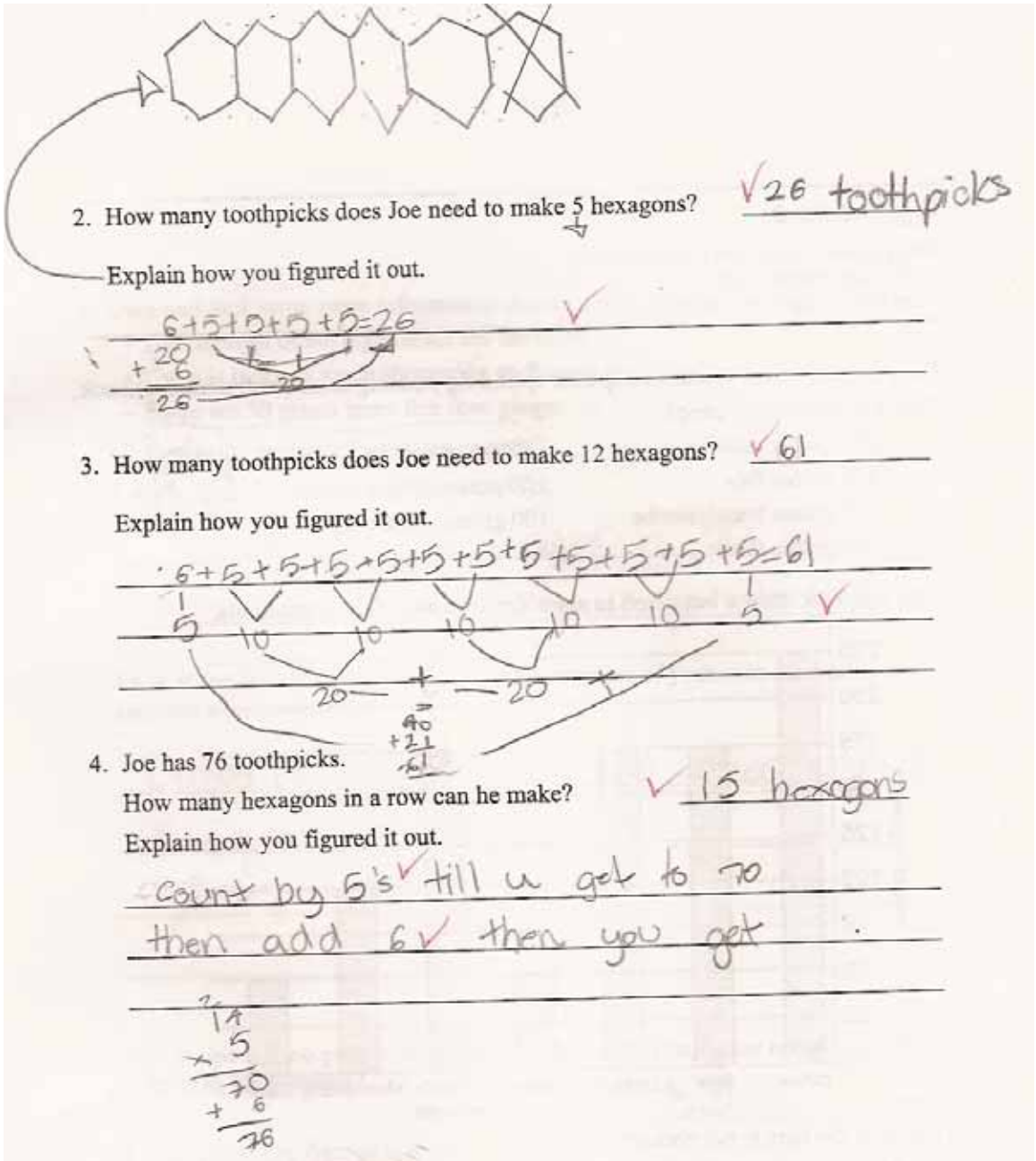
Explain how you figured it out.

15 X 6 = 90 − 14 = 76

I multiplied 15 by 6 which equals 90. I then subtracted 14 for the 14 shared toothpicks. 90-14=76 (I worked backwards).

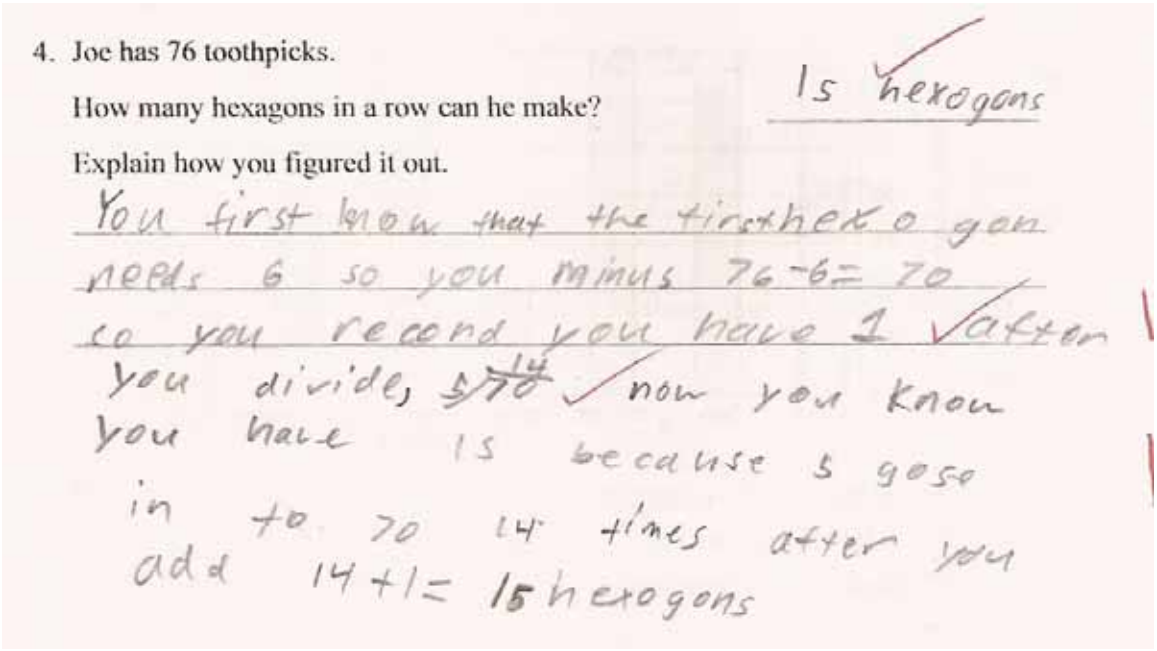
Student B is able to think about how the first term is different from the other terms and can use that strategy to solve the problem. Notice that the student knows that the 6 must be added back in to the pattern for part four and that the 6 represents and additional tile. This idea might be expressed symbolically as t=5(x-1)+6.

Student B



Student B2 also thinks about the 6+5+5+5. . . .However the student is able to generalize to a rule and use inverse operations in part 5 of the task. So if the rule is t=5(x-1)+6, the inverse would be x= [(t-6) /5] + 1.

Student B2



4. Joe has 76 toothpicks.

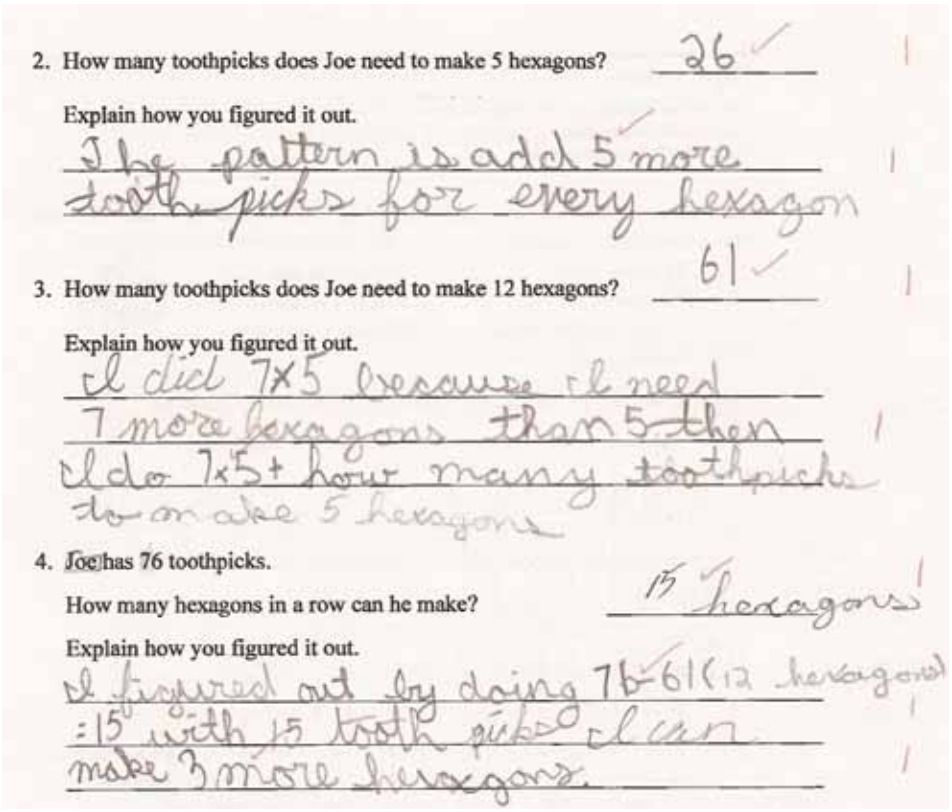
How many hexagons in a row can he make? 15 hexagons

Explain how you figured it out.

You first know that the first hexagon needs 6 so you minus 76-6= 70 so you record you have 1 after you divide, 5)70 14 now you know you have 15 because 5 goes in to 70 14 times after you add 14+1= 15 hexagons

Student C is able to use multiplicative thinking to see the number of groups of 5 that need to be added. Being able to use a unit, in this case a unit of 5, to measure up or down is an important step in developing proportional reasoning.

Student C



2. How many toothpicks does Joe need to make 5 hexagons? 26 ✓ 1

Explain how you figured it out.

I by pattern is add 5 more tooth picks for every hexagon ✓ 1

3. How many toothpicks does Joe need to make 12 hexagons? 61 ✓ 1

Explain how you figured it out.

I did 7x5 because I need 7 more hexagons than 5 then I do 7x5+ how many toothpicks to on ake 5 hexagons 1

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15 hexagons ✓ 1

Explain how you figured it out.

I figured out by doing 76-61(12 hexagons) =15 with 15 tooth picks I can make 3 more hexagons. ✓ 1

Student D is able to come up with the generalization of 5x+1 in a verbal form, and use that generalization to solve all the parts of the task.

Student D

Joe begins to make a table to show his results.

Number of hexagons in a row	1	2	3	4
Number of toothpicks	6	11	16	21

pattern x's 5, add 1

2. How many toothpicks does Joe need to make 5 hexagons? 26 toothpicks

Explain how you figured it out.

I first found a pattern, time's 5 add 1. so I did 5×5=25 then I added 1 to make 26.

3. How many toothpicks does Joe need to make 12 hexagons? 61 toothpicks

Explain how you figured it out.

I multiplied 12×5 which equals 60 then I added 1. The answer is 60 toothpicks

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15

Explain how you figured it out.

15 R1
5|76 I divided 76÷5= 15 R1 and then I checked by doing 15×5=75+1=76. so it goes back to the beginning

Student E makes a similar mathematical justification.

Student E

2. How many toothpicks does Joe need to make 5 hexagons? 26 toothpicks

Explain how you figured it out. 5 x 5 = 25, 25 + 1 = 26

each hexagon shares 1 side except for the first so if you multipy 5 x 5 you get 25 then add 1 toothpick because the first hexagon dosent share its side. So then you get 26 toothpicks. (25+1=26)

3. How many toothpicks does Joe need to make 12 hexagons? 61

Explain how you figured it out. 5 x 12 = 60, 60 + 1 = 61

each hexagon shares one side except for the first so if you multiply 5 and 12 you will get 60 then add 1 because the first hexagon dosent share its sides

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15

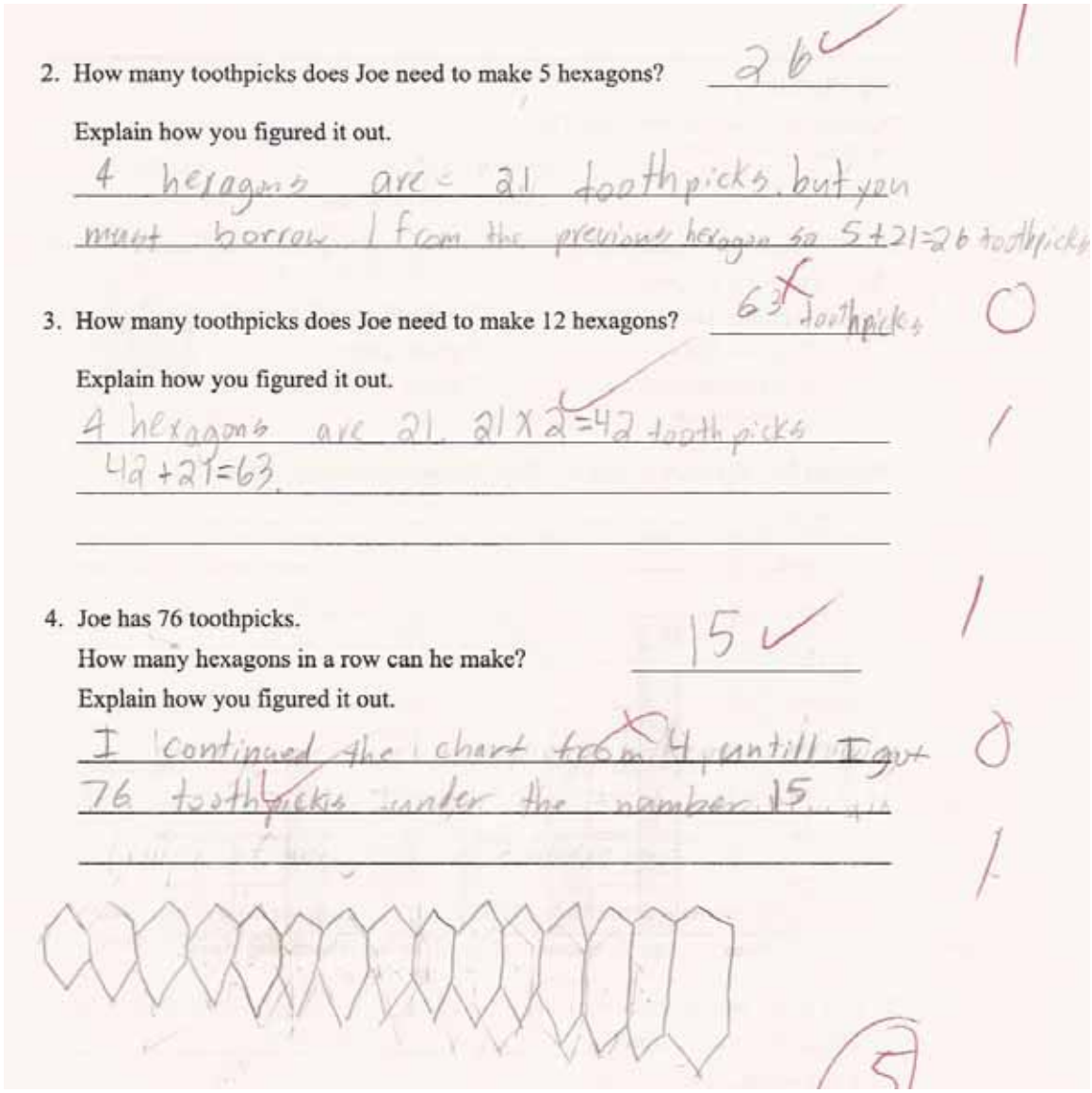
Explain how you figured it out.

The first hexagon dosn't share its side so I subtracted 1 from 76 and got 75 then I divided 75 by 5 because the rest of the hexagon do share their sides. 75 ÷ 5 = 15 and got 15

76 - 1 = 75
75 ÷ 5 = 15

Student F is able to complete a table by adding 5's to solve much of the task. However in part four, the student tries to use proportional reasoning if 4 hexagons are 21, then 21 x 3 should equal 12 hexagons. This reasoning does not work, because the constant is now included in the total 3 times instead of just once.

Student F



2. How many toothpicks does Joe need to make 5 hexagons? 26 ✓ 1

Explain how you figured it out.

4 hexagons are = 21 toothpicks. but you must borrow 1 from the previous hexagon so 5+21=26 toothpicks

3. How many toothpicks does Joe need to make 12 hexagons? 63 ✗ toothpicks 0

Explain how you figured it out.

4 hexagons are 21. 21 x 2 = 42 toothpicks 1

42 + 21 = 63.

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15 ✓ 1

Explain how you figured it out.

I continued the chart from 4, until I got 0

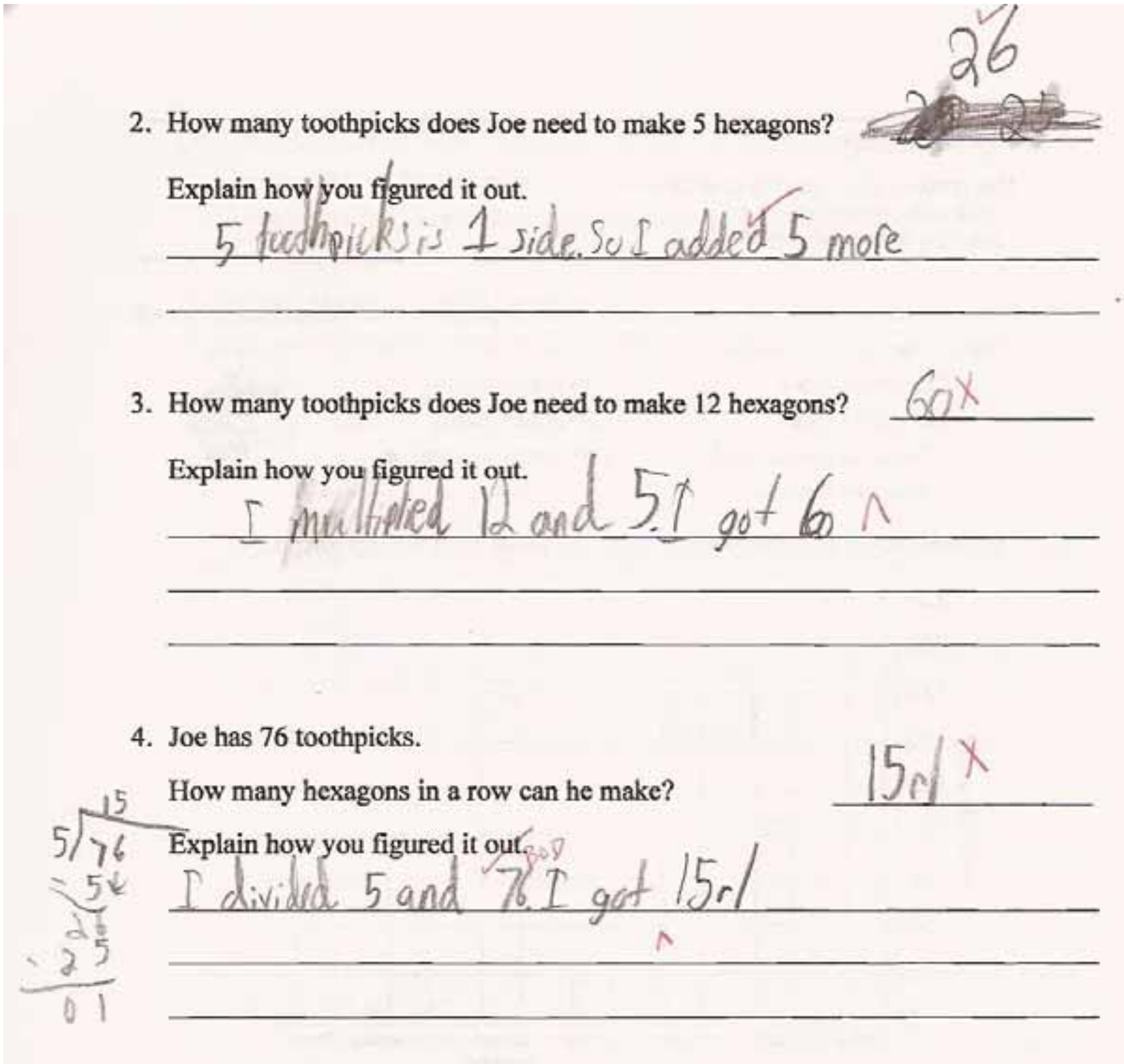
76 toothpicks under the number 15.

1

5

Student G understands that the growth rate is 5, but does not know how to add in the constant. In part three the student leaves out the constant, using a rule of 5x instead of 5x +1. In part four the student is unable to account for the extra "one".

Student G


26

2. How many toothpicks does Joe need to make 5 hexagons?

Explain how you figured it out.

5 toothpicks is 1 side. So I added 5 more

3. How many toothpicks does Joe need to make 12 hexagons? 60 X

Explain how you figured it out.

I multiplied 12 and 5. I got 60

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 15r1 X

Explain how you figured it out.

I divided 5 and 76. I got 15r1

By fifth grade, students should notice equal groups as they appear in a pattern. Students should start to feel comfortable measuring in units other than one, such as the "fiveness" represented in this pattern. Students should be able to start seeing equal groups as contexts for multiplication and division. Students at this grade level are striving for general rules about patterns, and some come up with verbal generalizations similar to the ones we want algebra students to express symbolically at later grades.

8% of the students were able to express a generalization in words equivalent to 5x+1 or 5(x-1)+6. 2% made generalizations that accounted for the number of overlaps. 4% of the students were able to bundle the 5's in groups (5 x3 or 5 x 6) and add it on to a previous quantity rather than doing a string of addition. 3.5% of the students could account for the difference in the first term (6+5+5+5…). 38% of the students used adding 5's or extending the table. 13% used a draw and count strategy correctly, while another 1% made errors using draw and count.

Incorrect strategies included 5% trying to use a times 5 strategy. Less than 1% used a times 6 strategy. 8% tried multiplying or adding parts of the table (6^{th} term x 2 = 12^{th} term) thus including the constant more than once. 1% had visual discrimination problems in their drawings. 2% had a rule of 5x + (wrong constant).

When looking at the papers for 5^{th} grade, I looked at the strategies for dealing with the inverse relationships in part 4 separately. 2.5% of the students could divide by 5 and then explain what the remainder meant. 12% understood that they needed to divide by 5, but couldn't explain the remainder. 8.5% of the students looked at the growth (76-61 or 76-26) and then were able to find the number of additional hexagons needed from the base number of hexagons. 12% were able to use generalizations ((76-6)/5 +1 or (76-1)/5). 6% tried to find the number for 76 hexagons instead of 76 toothpicks. 15% used draw and count for this part of the task, but for about 6%, this was the only part of the task where they reverted to a drawing strategy. 21% continued the table and 23% added on by 5's.

Fifth Grade

5th Grade Task 2 Hexagons in a Row

Student Task	Find a pattern in a sequence of diagrams and use the pattern to make predictions. Find the total number of iterations of hexagons that can be made when the total number of toothpicks is given.
Core Idea 3 Patterns, Functions, and Algebra	**Understand patterns and use mathematical models such as algebraic symbols and graphs to represent and understand quantitative relationships.** • Represent and analyze patterns and functions using words, tables, and graphs • Investigate how a change in one variable relates to a change in a second variable. • Find the results of a rule for a specific value • Use inverse operations to solve multi-step problems • Use concrete, pictorial, and verbal representations to solve problems involving unknowns

Mathematics in the task:
- Extend a geometric pattern
- Use a table
- Work backwards
- Understand the idea of a constant
- Recognize when a pattern is **not** proportional

Based on teacher observations, this is what fifth graders knew and were able to do:
- Add on to an existing pattern
- Recognize and verbalize a pattern (going up by 5's)
- Add on, multiply or divide by 5

Areas of difficulty for fifth graders:
- Multiplying by 6 instead of 5 (not noticing the overlap when hexagons are connected)
- Not seeing that the first hexagon has needs more toothpicks than the rest
- Seeing generalizable rules
- Drawing and counting accurately
- Dealing with the shared sides

Strategies used by successful students:
- Draw pictures
- Extended the table
- Seeing how the structural pattern of the hexagons grew and using that to form a rule

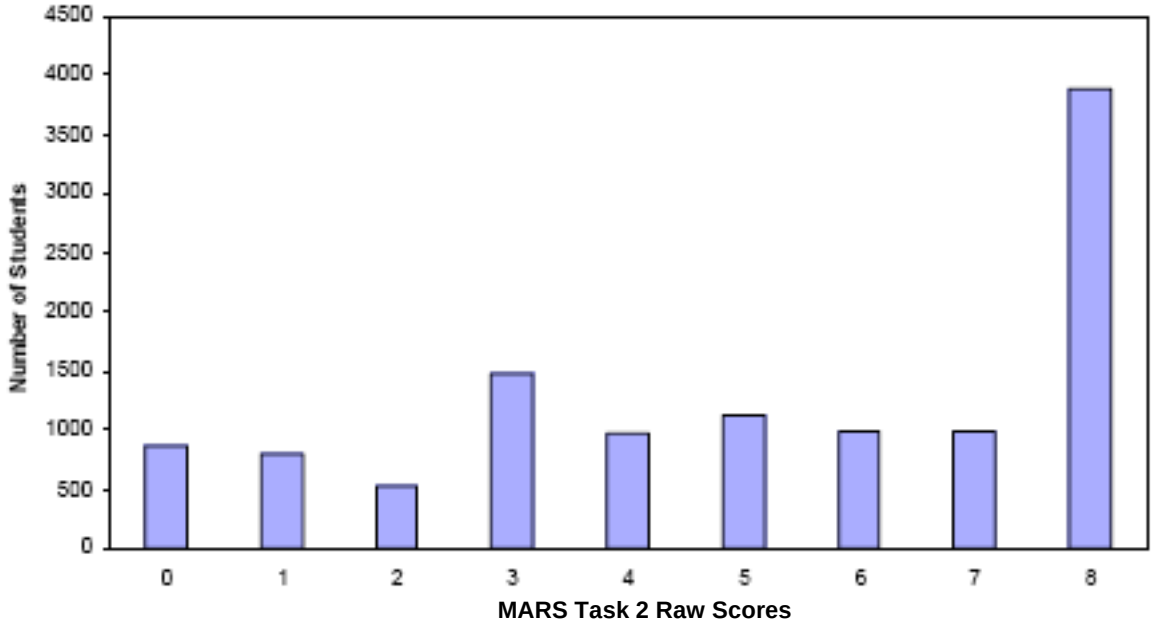
Task 2 – Hexagons in a Row

Mean: 5.14 StdDev: 2.71

Table 26: Frequency Distribution of MARS Test Task 2, Grade 5

Task 2 Scores	Student Count	% at or below	% at or above
0	858	7.4%	100.0%
1	793	14.2%	92.6%
2	531	18.8%	85.8%
3	1481	31.5%	81.2%
4	972	39.9%	68.5%
5	1127	49.6%	60.1%
6	987	58.1%	50.4%
7	991	66.6%	41.9%
8	3886	100.0%	33.4%

Figure 35: Bar Graph of MARS Test Task 2 Raw Scores, Grade 5



The maximum score for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Most students, 93%, could extend the pattern by filling in the table. Many students, 81%, could extend the pattern beyond the table to 5 hexagons and explain that the pattern is growing by 5 each time. More than half the students, 68%, could also do some of the thinking to solve for 12 hexagons, but they may have made a counting or calculation error. About half the students could also find the number of hexagons that could be made with 76 toothpicks. 33% could meet all the demands of the task including finding the number of toothpicks needed to make 12 hexagons in a row. 7% of the students scored no points on this task. All the students in the sample with this score attempted the task.

Hexagons in a Row

Points	Understandings	Misunderstandings
0	All the students with this score in the sample attempted the task.	Most students could read the diagrams and fill in 16 for 3 hexagons, Common answers for 4 hexagons: 20,24,22.
1	Students could use the diagrams to fill in the table for 3 and 4 hexagons.	Students had difficulty extending the pattern beyond the table. Some saw the "5" and thought the answer would be 25. Some thought about each hexagon having 6 sides, so they put 30. Some made calculation errors: 27,28,29, 32.
3	Students could extend the pattern to 5 hexagons and explain the pattern.	
4	Students could fill in the table, extend the pattern to 5 and explain how it grew, and do some of the work for part three or four with a counting or calculation error.	6% of the students knew the pattern was growing by 5, so they put 5 x 12 = 60. They forgot the extra 1 for the first hexagon. 5% thought that 4 x 3 is 12, so 21 x 3 = 63. They counted the first toothpick 2 extra times. 3% multiplied 12 times 6 (each hexagon has 6 toothpicks, ignoring the overlap).
6	Students could extend the table and work the pattern to 5 hexagons. Students could also work backwards from 76 toothpicks to find the number of hexagons in a row.	6% could subtract out the first hexagon (76-6) and divide the remainder by 5 (70/5=14), but they forgot to add back on the original hexagon to get 15. 3% thought the answer was 15 r 1. They couldn't explain what the remainder one represented. 4% tried to divide by 6.
8	Students could extend and describe a geometric pattern, using pictures, tables, and rules. Students could work backwards from the number toothpicks to the number of hexagons.	

Implications for Instruction

Students need more practice with spatial visualization and describing attributes of geometric shapes. They should be able to explain how a geometric pattern is formed and what changes as it grows. This focus on attributes helps students to move beyond counting strategies to find relationships about the pattern, which could lead to rules or generalizations for any number. Students should be able to notice that a pattern is growing by a set amount each time and then be able to use addition, continuing a table, or multiplication to continue the pattern.

Ideas for Action Research-Using Student Work to Process an Activity

In an action research group, teachers looked at a class set of student papers. The teacher had given one set of students the hexagon task as it appears on the 2006 exam, For the other half of the students, the teacher eliminated the table but asked the students the same questions. How many toothpicks are needed to make 3 hexagons? How many toothpicks are needed to make 4 hexagons? The second page of the task was the same for both groups of students. The conjecture was that students without the table would use different strategies or ways of thinking about the pattern. You might try this to see what you notice. What conjectures do you have about how the table supports students' thinking? How do you think taking away the table might effect student thinking?

The teachers made a table like this to categorize their results (incorrect strategies are in italics)

Students with a Table		**Students without a Table**	
Strategy for #2	Number of students	**Strategy for #2**	Number of students
Draw		Draw	
Add 5		Add 5	
1st is 6, extras are 5		1st is 6, extras are 5	
+ 6 minus 1		+ 6 minus 1	
Multiply by 6		*Multiply by 6*	
Strategy for #3	Number of students	**Strategy for #3**	Number of students
Continue table		Make a table	
Draw and count		Draw and count	
Add by 5's		Add by 5's	
Add on 26+ (7x5)		Add on 26+ (7x5)	
12x5 +1		12x5 +1	
6 +(5x11)		6 +(5x11)	
(31x2) -1		(31x2) -1	
(12 x 6)-11		(12 x 6)-11	
4x + (x+1)		4x + (x+1)	
(12 x 5)-11		*(12 x 5)-11*	
Multiply by 6		*Multiply by 6*	
(31 x 2)		*(31 x 2)*	
12 x7		*12 x7*	
Strategy for #4	Number of students	**Strategy for #4**	Number of students
Draw			
Add 5			
5x+1			
76-6=70 70/5=14 14+1=15			
(76-61)=15 15/5=3 12+3=15			
Divide by 4			
Divide by 6			

Next teachers discussed what they thought was the mathematical story of the problem and thought about how to process the big ideas with this class using student work. You might want to try this process with your own student work or use the examples below to process the activity. You might also see the notes used by the teacher and think if there are different questions you might ask. The idea is to show part of thinking and have all students try to decide if it makes sense or not. This helps students to re-engage in the mathematics and look at the mathematics from a different perspective.

First Student

2. How many toothpicks does Joe need to make 5 hexagons? 25 ✓ toothpicks 1

Explain how you figured it out.

I found out that after one more hexagon will give you five more toothpicks. ✓ 1

3. How many toothpicks does Joe need to make 12 hexagons? 61 ✓ toothpicks. 1

Explain how you figured it out.

Just add five more toothpicks for every hexagon. ✓ 1

4. Joe has ~~76~~ 75 toothpicks.

How many hexagons in a row can he make? 15 ✓ hexagons 1

Explain how you figured it out.

7x5+1=76 will give you the answer. ✓ 1

X 0

(1) Where is the 5? where is the 1?

How would you use this to figure out 40 hex.?

8

Hexagons in a Row Test 5

1/2

7

Student 2

2. How many toothpicks does Joe need to make 5 hexagons? 26 ✓

Explain how you figured it out.

For more than one hexagon, each one needs five (with an exception of the first one which needs six). ✓ 1

3. How many toothpicks does Joe need to make 12 hexagons? 61 ✓ 1

Explain how you figured it out. 55 + 6 = 61

6+(5x11)=61 ✓

(2) Where is the 6? Where is the 5? (in the drawing)

What would go where the "11" is for 40 hex.? How do you know?

4. Joe has 76 toothpicks.

How many hexagons in a row can he make?

Explain how you figured it out.

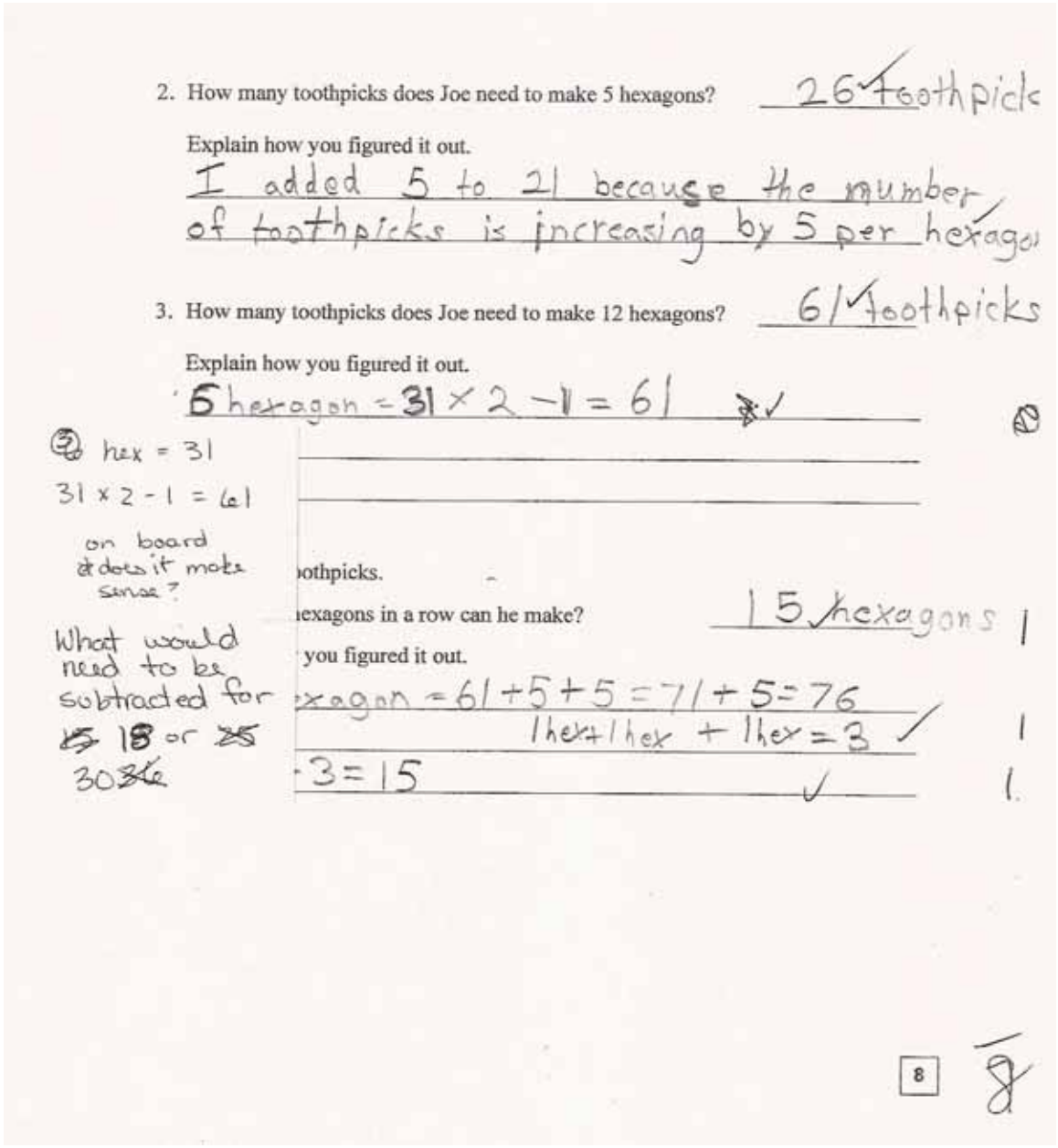
76-6=70÷5=14+1=15 ✓

X

compare to ? x5+1

One says +6 other +1 why do they both work?

Student 3



2. How many toothpicks does Joe need to make 5 hexagons? 26 toothpicks

Explain how you figured it out.

I added 5 to 21 because the number of toothpicks is increasing by 5 per hexago

3. How many toothpicks does Joe need to make 12 hexagons? 61 toothpicks

Explain how you figured it out.

5 hexagon = 31 × 2 − 1 = 61

hex = 31

31 × 2 - 1 = 61

on board & does it make sense?

oothpicks.

iexagons in a row can he make? 15 hexagons

you figured it out.

exagon = 61 + 5 + 5 = 71 + 5 = 76

1hex + 1hex + 1hex = 3

-3 = 15

What would need to be subtracted for ~~15~~ 15 or ~~25~~ 30 ~~36~~

8

Student 4

4n + (n+1) + diagram

2. How many toothpicks does Joe need to make 5 hexagons? 1

Explain how you figured it out.

I drew another hexagon next to the four hexagons already there. ✓ 1

3. How many toothpicks does Joe need to make 12 hexagons? 61 ✓ 1

Explain how you figured it out.

I multiplied 4 with the amount of hexagons, 4×12=48, and added that, for the number 1 more the number of hexagons. 48+13=61 ✓ 1

4. Joe has 76 toothpicks.

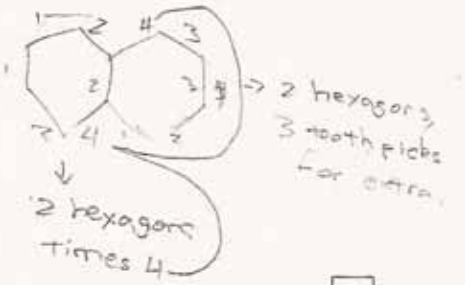
How many hexagons in a row can he make? 19 X 0

Explain how you figured it out.

I used the first part of the way I used above. The 4 came from this pattern: X 0

Use whole statement from #3?
Does it work?

76÷4=19



8

For the next part the teacher wants to put up 2 strategies, those for Student 5 and 6 and have the students compare. Which makes sense? Why?

Student 5

2. How many toothpicks does Joe need to make 5 hexagons? 26 ✓ 1

Explain how you figured it out.

I figured it out in that if 4 hexagons is 21 I add √5 because one side of the after hexagon is filled 1

3. How many toothpicks does Joe need to make 12 hexagons? 49 X 0

Explain how you figured it out.

12×5=60 but you have to subtract 11 toothpicks X 0
because it has a pattern. If 2 hexagons are
made one side is filled so you subtract one less.

4. Joe has 76 toothpicks.

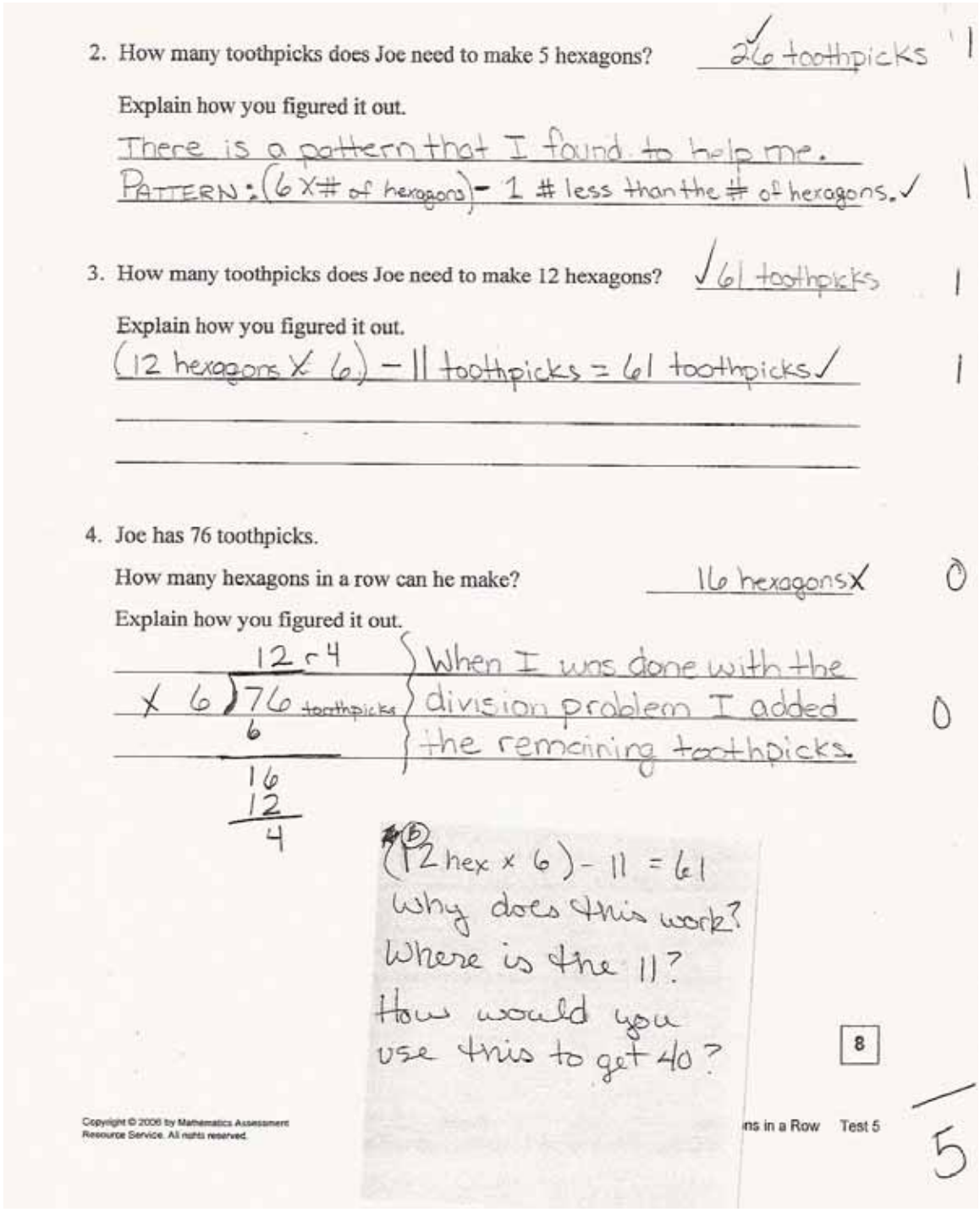
How many hexagons in a row can he make? 7 X 0

Explain how you figured it out.

12×7=84 but 84-6=78 so 7 is the answer. X 0
If it was six it would be too less.

(B) (A) use A + B
which one works?

Student 6



2. How many toothpicks does Joe need to make 5 hexagons? 26 toothpicks ✓ 1

Explain how you figured it out.

There is a pattern that I found to help me.
PATTERN: (6 X # of hexagons) - 1 # less than the # of hexagons. ✓ 1

3. How many toothpicks does Joe need to make 12 hexagons? ✓61 toothpicks 1

Explain how you figured it out.

(12 hexagons X 6) - 11 toothpicks = 61 toothpicks ✓ 1

4. Joe has 76 toothpicks.

How many hexagons in a row can he make? 16 hexagons X 0

Explain how you figured it out.

12 r 4
X 6) 76 toothpicks
6
16
12
4

When I was done with the division problem I added the remaining toothpicks. 0

(12 hex x 6) - 11 = 61
Why does this work?
Where is the 11?
How would you use this to get 40?

8

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ns in a Row Test 5

5

- How did this discussion help to re-engage students in the mathematics? Do you think some of them changed their thinking as the discussion progressed or might use a different strategy next time they have a pattern problem?
- How did the discussion help to pull out the important mathematics of the task?
- What further ideas still need to be discussed?

Performance Assessment Task

Hexagons in a Row
Grade 5

This task challenges a student to use knowledge of number patterns and operations to identify and extend a pattern. A student must be able to describe the changing pattern in ordered pairs using a table. Must be able to understand the relationship between two variables and relationships between operations to extend the pattern given any part of the relationship. A student must be able to use knowledge of patterns to evaluate and test a conjecture about how a pattern grows. A student must be able to model a problem situation with objects and use representations such as tables and number sentences to draw conclusions. A student must be able to explain and quantify the growth of a numerical pattern.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core

points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
5	2006	8	4	69%