Graphs

This problem gives you the chance to:
• relate equations, descriptions and graphs of a relationship

On the opposite page are four graphs.

Below are four descriptions and four equations.

Choose the description and the equation that fits each graph.

Write the correct description and the correct equation under each graph.

Descriptions:

x is the width and y is the length of a rectangle with a constant perimeter.

x is the radius and y is the area of a circle.

x is the length and y is the width of a rectangle with a constant area.

x is the radius and y is the circumference of a circle.

Equations:

$y = \frac{k}{x}$

$y = kx$

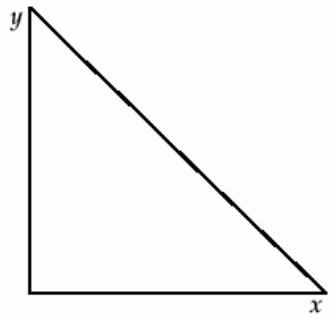
$y = kx^2$

$y = k - x$

In each equation, k is a fixed number.

1. Explain how you made your choices.

Graph A

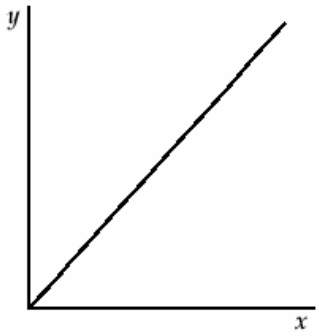


Description _____

_____

Equation _____

Graph B

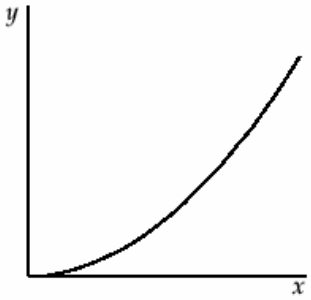


Description _____

_____

Equation _____

Graph C

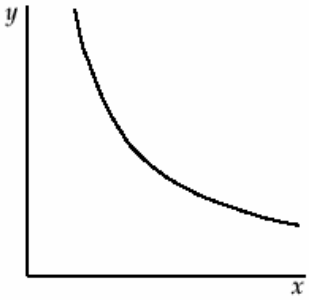


Description _____

_____

Equation _____

Graph D



Description _____

_____

Equation _____

Graphs	Test 9 Rubric	
The core elements of performance required by this task are: • relate equations, descriptions and graphs of a relationship Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct explanations such as: Graphs A and B are straight lines. Graphs C and D are curves. Graphs B and C pass through the origin. Graph D: the axes are asymptotes. Any correct statement about the mathematics 1 point.	1	max 1
2. Correct allocation of descriptions and equations to graphs. **Graph A** x is the width and y is the length of a rectangle with a constant perimeter. $y = k - x$ **Graph B** x is the radius and y is the circumference of a circle. $y = kx$ **Graph C** x is the radius and y is the area of a circle. $y = kx^2$ **Graph D** x is the length and y is the width of a rectangle with a constant area. $y = \frac{k}{x}$ Max 4 points for correct descriptions. Max 3 points for correct equations.	1 1 1 1 1 1 1 1	max 7
Total Points		**8**

Looking at Student Work – Graphs

Out of 165 students in the sample, no student scored above a 5. No student showed evidence of trying to write out an equation to match the descriptions given in the problem. No student in the sample showed evidence of making a t-chart to put in values for x and y to match and help graph the equations given in the problem.

Student A is a typical 5 paper. The student could not articulate using mathematics to think about how to match descriptions or equations to their graphs. The student is able to correctly identify all 4 equations, but struggles with understanding the descriptions.

Student A

Descriptions:

x is the width and y is the length of a rectangle with a constant perimeter.

x is the radius and y is the area of a circle.

x is the length and y is the width of a rectangle with a constant area.

x is the radius and y is the circumference of a circle.

Equations:

$y = \frac{k}{x}$

$y = kx$

$y = kx^2$

$y = k - x$

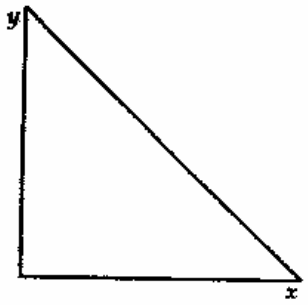
In each equation, k is a fixed number.

1. Explain how you made your choices.

I honestly didn't understand this so I made educated guesses

Student A, part 2

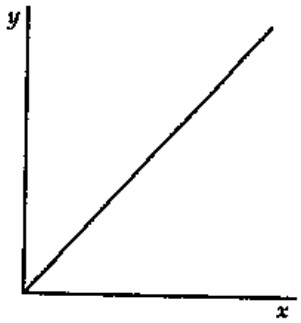
Graph A



Description x is the width and y is the length of a rectangle with a constant perimiter.

Equation $y = k - x$ ✓

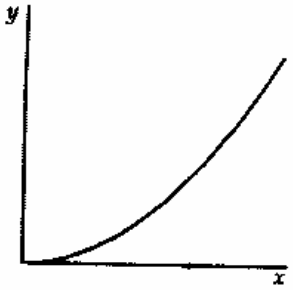
Graph B



Description x is the radius y is area of a circle

Equation $y = kx$ ✓

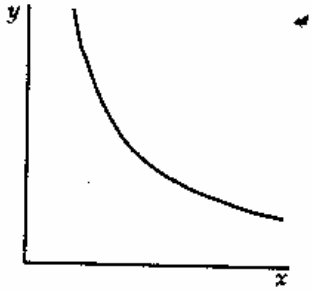
Graph C



Description x is the radius and y is the circumference of a circle

Equation $y = kx^2$ ✓

Graph D



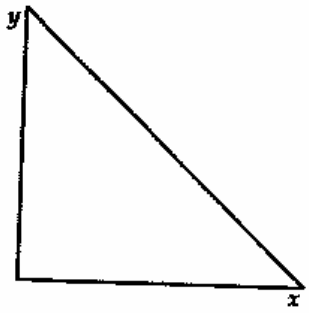
Description x is the length and y is the width of a rectangle with a constant area

Equation $y = \frac{k}{x}$ ✓

Student B also states, "I guessed. I've never seen this before." The student is able to correctly guess all the equations, but misses all the descriptors.

Student B

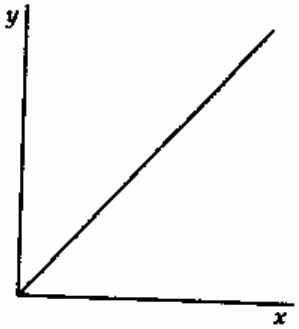
Graph A



Description x is the width & y is the length of a rec.

Equation ~~$y = kx^2$~~ $y = k - x$

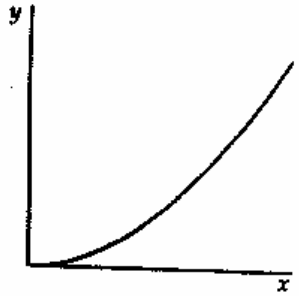
Graph B



Description x is the length y is the width

Equation $y = kx$

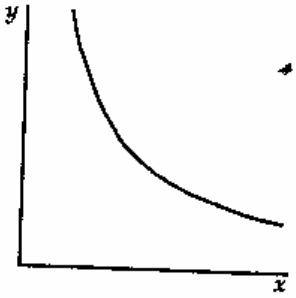
Graph C



Description x is radius y is circumference

Equation ~~$y = x$~~ $y = kx^2$

Graph D



Description y is radius x is circumference

Equation $y = \frac{k}{x}$

Student C expresses the idea of thinking about whether the equations are going to be increasing or decreasing. In examining the error patterns, it seems many students were able to make this level of distinction between the equations and the graphs. However this only gave them a 50% chance of being right. It also does help students to think about the matching descriptors to the graph.

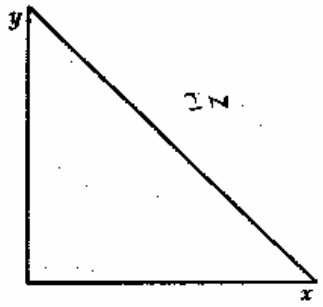
Student C

In each equation, k is a fixed number.

1. Explain how you made your choices. X D

FIRST I WROTE whether it was INVERSE or direct. THEN I thought of two NUMBERS and put it in the EQUATION FOR the description I thought the description through and WHICH GRAPH it would apply to BOTH

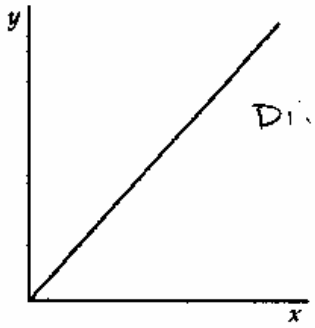
Graph A



Description X is the length and Y is the width of a rectangle with a X

Equation $Y = \frac{K}{X}$ X

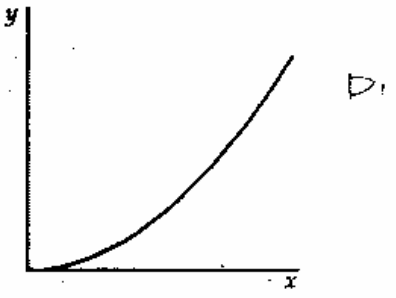
Graph B



Description X is the width X Y is the length of a rectangle w/ a CONSTANT PERIMETER.

Equation $K-X=Y$ X

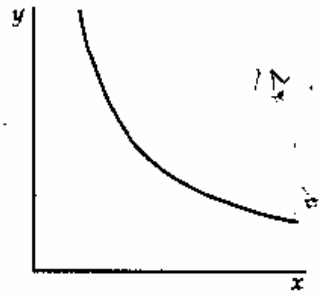
Graph C



Description X is the radius and Y is the area of a circle ✓

Equation $KX^2=Y$ ✓

Graph D



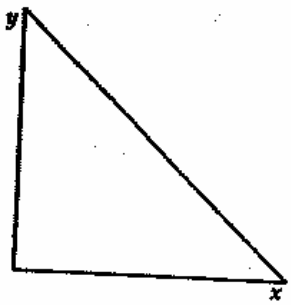
Description X is the radius X and Y is the CIRCUMFERENCE of a circle

Equation $Y=K-X$ X

Some students, like D and E, did not use the equations and/ or descriptions provided in the prompt. Student D tries to describe the shape of the graphs and make up equations. Student E also describes the shape of the graph.

Student D

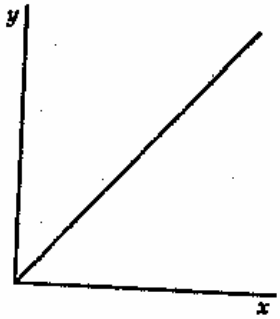
Graph A



Description curved horizontal up and down

Equation y = x

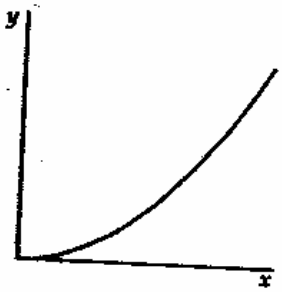
Graph B



Description Strait up to the right.

Equation y = x

Graph C



Description curved down left & right

Equation x = y

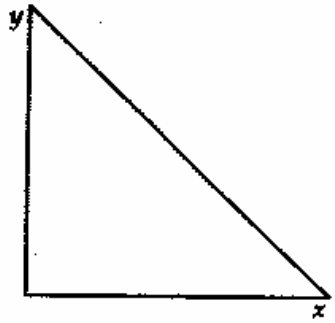
Graph D

Description curved left and right

Equation y = x

Student E

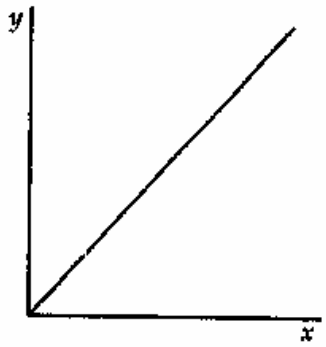
Graph A



Description It is a triangle X

Equation y = kx X

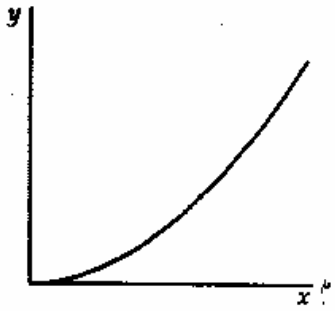
Graph B



Description the line increase quicky X

Equation $y = \frac{k}{x}$ X

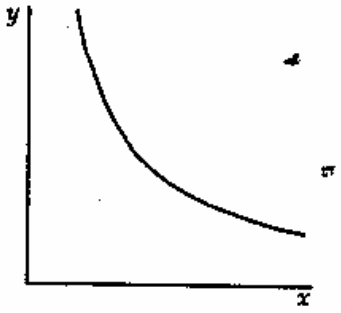
Graph C



Description the line increase gradually X ✓

Equation $y = kx^2$ ✓

Graph D

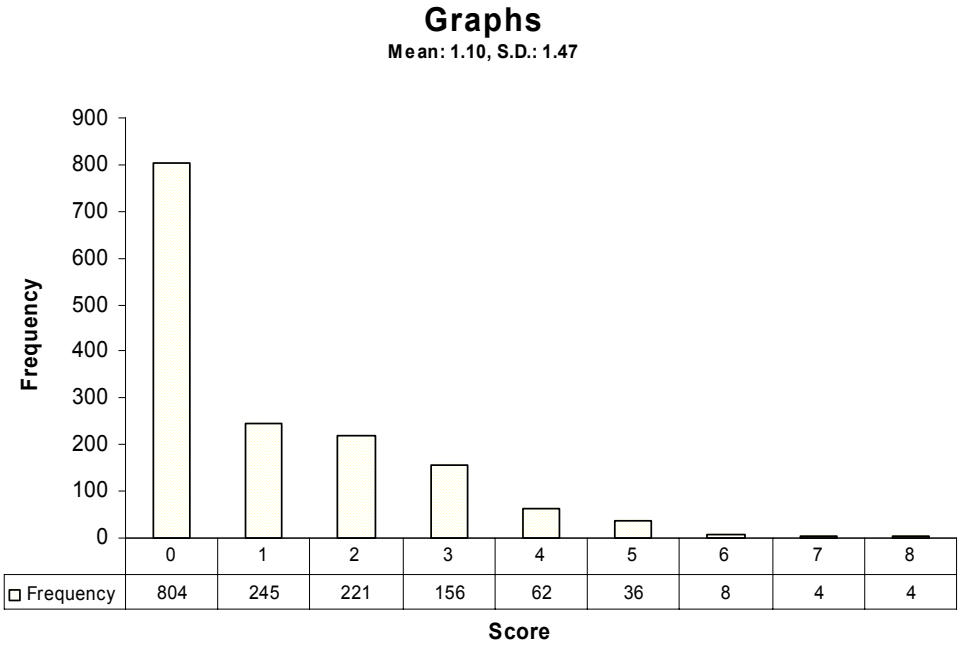


Description the line decrease slowly. X

Equation y = k - x X

Teacher Notes:

Frequency Distribution for each Task – Grade 9
Grade 9– Graphs



Score:	0	1	2	3	4	5	6	7	8
% < =	52.2%	68.1%	82.5%	92.6%	96.6%	99.0%	99.5%	99.7%	100.0%
% > =	100.0%	47.8%	31.9%	17.5%	7.4%	31.9%	1.0%	0.5%	0.3%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 3 points.

About half the students could give a description for either graph A or C or give an equation for B or C. A little less than 20% of the students could correctly identify a couple of the descriptions and at least one equation. Less than 1% could match all the descriptions and equations. Over 52% of the students scored no points on this task. 32% of them attempted the task. 46% did not attempt this task, but were willing to attempt the task following graphs. 22% did not attempt this task or the final task of the exam.

Graphs

Points	Understandings	Misunderstandings
0	Only 32% of the students with this score attempted the task.	Students in the entire sample did not seem to have a strategy for connecting the equations or descriptions to the graphs. Students seemed to imply in part 1 that they guessed. There is an underlying belief that they should just know it without doing any work. Math answers are quick.
1	The easiest answers for students were the descriptions for A and C or the equation for C.	Students did not show evidence of trying to write equations for the descriptions or of making t-charts of values for x and y to help match equations to graphs.
3	Students with this score could generally match two descriptions and one equation. Their correct descriptions were not necessarily the same as their correct equation.	Some students seemed to know which equations would create graphs with positive or negative slopes, but then did not necessarily make the appropriate guess between the two choices.
5	Students could generally identify all of the correct equations and only two correct descriptions.	
6/7		No students in the sample had this score.
8	Less than 1% of the students could describe correctly how they matched descriptions and equations to graphs and then successfully match all the descriptions and equations to the 4 given graphs.	The equations and descriptions about circumference and constant area were most difficult for students to think about.

Questions for Reflection on Graphs

- How is graphing taught in your classroom?
- Do students have the opportunity to make t-charts for x and y values to help plot graphs?
- Do students have the opportunity to talk about how the equations for curves are different from the equations for straight lines?
- What type of student work would you have expected to see from students to help solve this problem or as a rationale for part 1? Why do you think students tried to guess instead of using mathematics to work out the correct solutions?
- Why do you think so few students were willing to even attempt the problem when all the answers were given?
- Why do you think students didn't see a connection between the graphs in the problem and the types of graphs in their textbooks?
- What types of problems or investigations have students done this year that might have developed the type of logic or strategies needed to solve this task? What other resources do you have that might have these types of problems or investigations?

Look at the papers for your best student. Was there anything significant between the types of responses they gave and those responses for other students in the classroom?

Teacher Notes:

Implications for Instruction:

Students need to be able to make connections between multiple representations of the same idea. They should have frequent opportunities to work with situations, turn those situations into equations, and make graphs from the equations. In reading the sample, students did not show evidence of trying to turn the words in the descriptions into equations. No student in the sample attempted to make a t-chart of values for any of the equations to help them match the equation to a graphic representation. Learning skills without being able to make connections is not helpful in transferring knowledge to any application outside the classroom. Students working in elementary algebra should be quite fluid with this level of making connections.

Teacher Notes:

Performance Assessment Task

Graphs (2004)
Grade 9

This task challenges a student to use knowledge of functions to match verbal descriptions of a context with equations and graphs. A student must be able to construct an argument based on properties of graphs and equations to justify matches.

Common Core State Standards Math - Content Standards

High School – Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal descriptions of the relationship. *Key features include: intercepts, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums, symmetries and behavior; and periodicity.*

F-IF.5 Relate the domain of a function to its graph and where, applicable, to the quantitative relationship it describes. *For example, if the function h (n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxim, and minima

F-IF 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of

interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2004	8	3	18%