Fair Game?

This problem gives you the chance to:
• use probability to judge the fairness of a game

Chris and Jack are playing a board game with a fair coin and a fair number cube numbered 1 to 6.

They take turns tossing the coin and the number cube.

Then they figure out the score.

> If the coin lands on heads, the score is twice the number on the number cube.
> If the coin lands on tails, the score is two more than the number on the number cube.

1. Complete this table of possible scores.

Number on the number cube

Coin		1	2	3	4	5	6
	Heads	2	4	6			
	Tails	3	4	5			
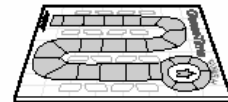
2. If the score is a prime number, Chris moves two squares on the board.

If the score is not a prime number, Jack moves one square on the board.

What is the probability of getting a score that is a prime number? _____
Show your work.

Fair Game?   Test 7: Form A

3. Chris and Jack play a game in which there are 12 trials.

How many squares would you expect Chris to move? _____

How many squares would you expect Jack to move? _____

4. Is this game fair? _____
Explain your answer.

_____

_____

_____

_____

8

Fair Game?   Test 7: Form A

Fair Game?

The core elements of performance required by this task are: • use probability to judge the fairness of a game Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Correctly completes the table: **Number on the number cube** _Table:_ Coin — Heads: 1→2, 2→4, 3→6, 4→8, 5→10, 6→12 Coin — Tails: 1→3, 2→4, 3→5, 4→6, 5→7, 6→8 *Allow 1 point for each 2 correct values.*	3 × 1	3
2. Gives a correct answer as: $\frac{4}{12}$ Shows that there are 12 possible scores, only 2, 3, 5, 7 are prime. *Accept $\frac{4}{12}$ for work.*	1 1	2
3. Gives correct answers as: Chris moves **8** squares. Jack moves **8** squares.	1 1	2
4. Gives correct answer as: Yes, the game is fair. and Explains that in the long term, both players move the same number of squares.	1	1
Total Points		**8**

The table referenced in item 1:

		1	2	3	4	5	6
Coin	Heads	2	4	6	8	10	12
	Tails	3	4	5	6	7	8

Fair Game

California State Standards and Textbooks expect mastery of probability in 6th grade and retention of those skills to last until the high school exit exam. However, results show that many students need more than one year to learn this concept.

Student A shows a nice job of listing all the outcomes in the sample space and clearly defining the prime numbers in part 2. Student A also shows facility with multiplying fractions and applying the rules of the game in part 3. The student does a good job of explaining the relationship between the rules for spaces moved and the differing probabilities to make the game fair.
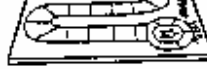
Student A

1. Complete this table of possible scores.

$\bigcirc$ = prime #

Number on the number cube

		1	2	3	4	5	6
Coin	Heads	②	4	6	8	10	12
	Tails	③	4	⑤	6	⑦	8

2. If the score is a prime number, Chris moves two squares on the board.

If the score is not a prime number, Jack moves one square on the board.

What is the probability of getting a score that is a prime number? $\frac{4}{12}$ or $\frac{1}{3}$ of get

Show your work.

P – (H,2) p–(T,3)
(H,4) (T,4)
(H,6) p–(T,5)
(H,8) (T,6)
(H,10) p–(T,7)
(H,12) (T,8)

$\frac{4 \text{ prime}}{12 \text{ total}}$ ✓

Page 6 Fair Game? Test 7: Form

3. Chris and Jack play a game in which there are 12 trials. $\frac{4}{12} \times \frac{12}{1} = \frac{48}{12} = 4 \cdot 2 = 8$

How many squares would you expect Chris to move? _____ 8 moves ✓

How many squares would you expect Jack to move? _____ 8 moves ✓

$12\overline{)96}$ $\frac{8}{96}$ $\frac{12}{12} - \frac{4}{12} = \frac{8}{12} \times \frac{12}{1} = \frac{96}{12} =$

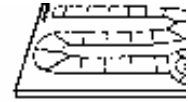4. Is this game fair? _____ It is fair ✓

Explain your answer.

The game is fair because in 12 trials, both people move 8 places. Even though Chris' chance of getting a prime number is lower than not a prime number, he moves 2 places. That makes the game fair.

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Student B also demonstrates a complete knowledge of probability. The only error in her thinking is to forget that 2 is a prime number in part 2 of the task. This error in finding probability affects further calculations.

Student B

Number on the number cube

		1	2	3	4	5	6
Coin	Heads	2	4	6	8	10	12
	Tails	3	4	5	6	7	8

2. If the score is a prime number, Chris moves two squares on the board.

If the score is not a prime number, Jack moves one square on the board.

What is the probability of getting a score that is a prime number?
Show your work.

$$\frac{3}{12} = \frac{1}{4}$$

3. Chris and Jack play a game in which there are 12 trials.

How many squares would you expect Chris to move?

6 squares

How many squares would you expect Jack to move?

9 squares

4. Is this game fair?
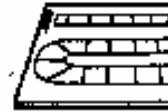Explain your answer.

No it's 0

not because there are only 3 prime numbers and nine not prime numbers and so even though Chris moves 2 squares on a prime, after 12 turns he will be behind 3 squares and after 24, he'll be behind 6 squares and so on.

More than 1/3 of the students looked at only the numbers on the number cube when calculating probabilities instead of the outcomes. Depending on whether they included 2 as a prime number their probabilities were either 50% or 4/6. Few of these students related their incorrect probabilities to the rules of the game and calculations for Chris and Jack in part 3 of the task. See the work of Student C.

Student C

1. Complete this table of possible scores.

Number on the number cube

Coin		1	2	3	4	5	6
	Heads	2	4	6	8	10	12
	Tails	3	4	5	6	7	8

2. If the score is a prime number, Chris moves two squares on the board.

If the score is not a prime number, Jack moves one square on the board.

What is the probability of getting a score that is a prime number? Show your work.

$$X \frac{3}{6} = \frac{1}{2}$$

1 3 5 2 4 6

3 out of 6 = $\frac{1}{2}$

3. Chris and Jack play a game in which there are 12 trials.

How many squares would you expect Chris to move?

How many squares would you expect Jack to move?

$$X \frac{1}{2} = 6 X$$

$$X \frac{1}{2} = 6 X$$

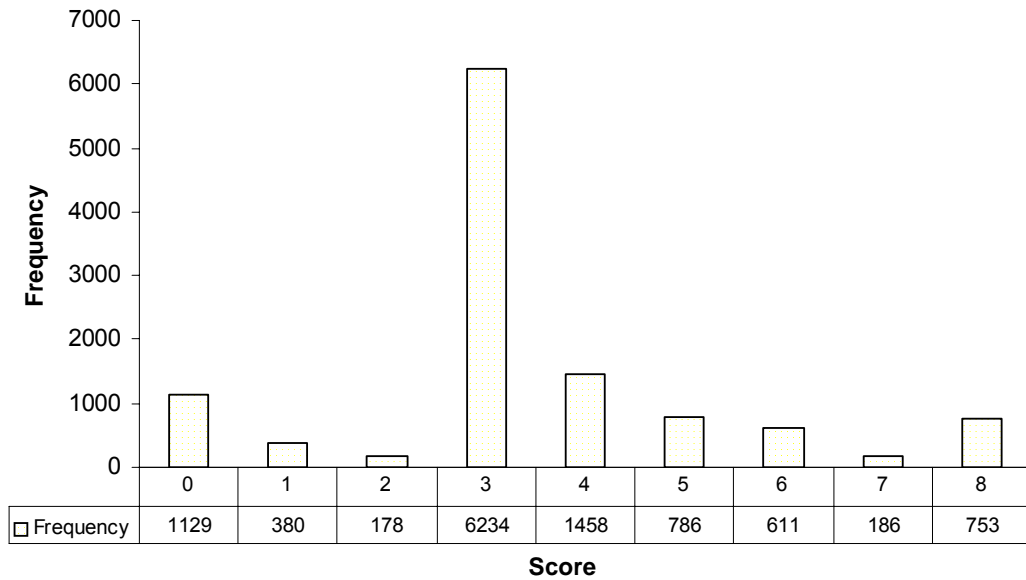4. Is this game fair?
Explain your answer.

yes

Because each one of them move an equall amount of spaces so their for the game is fair

Teacher Notes:

Fair Game?
Mean, 3.43, S.D.: 1.88

Score:	0	1	2	3	4	5	6	7	8
% < =	9.6%	12.9%	14.4%	67.6%	80.1%	86.8%	92.0%	93.6%	100.0%
% > =	100.0%	90.4%	87.1%	85.6%	32.4%	19.9%	19.9%	8.0%	6.4%

The maximum score available for this task is 8 points.
The cut score for a level 3 response is 4 points.

Many students (about 85%) could apply the two different rules to fill out the table of outcomes. Less than 20% could find the number of primes and correctly calculate the probability for getting a prime. Less than 10% of the students could meet all the demands of the task. 10% of the students scored no points on this task. 5% of the students appeared to have run out of time, because there is no work on task 4 or task 5.

Fair Game?

Points	Understandings	Misunderstandings
0	About half the students with this score attempted the problem. The other half appear to have run out of time.	Students with this score did not read the rules carefully for getting heads and tails. Many just made a pattern of even numbers and odd numbers or going up one every time.
1	Students could get the table right for heads.	Students did not understand the rules for changing the number of the cube is a tails was tossed.
3	Students could fill out the table correctly.	About 22% of the students calculated the probability of based on the numbers on the cube instead of the numbers in the table. Most also forgot that 2 was a prime number, so their probability was 1/2 or 50%. Another 14% used the table but forgot to count 2 as a prime number and therefore had 1/4 or 3/12 as their probability. 14% of the students wrote probabilities as whole numbers.
4	Students could fill out the table and explain why the game is fair in part 4.	
5	Students could fill out the table and show correct thinking for finding the probability in part 2.	Students could not use their probabilities (right or wrong) to find theoretical probabilities and then correctly apply the rules for moving to determine the outcome of the game. There was not a consistent pattern for these errors. In some cases the students just added two to one persons score, doubled both scores, thought the total moves should be 12, or other strategies.
8	Students could fill out the table using different rules for heads and tails, find the probability of getting a prime number, and find the theoretical probability of each student moving in 12 turns and apply the different rules for their movement. Students could clearly explain why the game was fair even though the 2 players had different probabilities for getting to move.	

Based on teacher observations, this is what seventh grade students seemed to know and be able to do:
- Follow rules to fill out a table

Areas of difficulty for seventh graders, seventh grade students struggled with:
- Identifying 2 as a prime number
- Calculating probabilities from a given set of outcomes or recognizing the difference between the random number generators and the outcomes (*many students thought things were 50-50 because there are two sides on a coin or three of the 6 numbers on the cube are prime*)
- Using probability to find expected outcomes in a game situation. Giving a good mathematical argument for why a game is fair combining probabilities and rules of the game

Questions for Reflection on Fair Game?

Look carefully at student work, how many of your students could:
Fill out the table?
How many gave the probability in part 2 as:

1/3	1/5	1/4	4/6	Whole numbers	Other?

Did your students have trouble multiplying the fractions to find the expected moves in part 3? Did they apply the rules for how students moved correctly?

What kind of program does your school have in place for students who do not master probability as 6th graders? Have you discussed this issue at department meetings? Do other teachers know the importance of this topic on the high school exit exam?

What types of probability activities have students had this year? What are some of your favorite activities that might help students who are still struggling with probability?

Teacher Notes:

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Instructional Implications:

Students need more experiences with games and probability. Students do not know how to combine two events, like tossing a coin and a number cube, to find the probability. They confuse the random number generators with the outcomes. While students can complete tables showing what is possible, they have difficulty determining how to answer questions based on that table. When it is possible to move a different number of spaces for each player, they don't know how to use that information to determine who has a better chance of winning. Students do not understand that probability or expected outcomes are determined by a large number of events. To compare whether a game is fair, the outcomes of several games needs to be considered. Students also needed to know mathematical vocabulary like prime number, fair, and trial to work this problem successfully.

Teacher Notes:

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Performance Assessment Task
Fair Game?
Grade 7
This task challenges a student to use understanding of probabilities to represent the sample space for simple and compound events. A student must use information about probabilities to estimate probability of future events and construct an argument about the fairness of a game.

Common Core State Standards Math - Content Standards

Statistics and Probability
Investigate chance processes and develop, use, and evaluate probability models.
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times,*

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of discrepancy.
  b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.
  a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
  b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space, which compose the event.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2003	8	4	32%