

LINDA FISHER: When I was looking at it, on the very first problem, remember I asked why they were going to spend so much time on the $\frac{6}{9}$ ths and the $\frac{2}{3}$ ds, because kids in that classroom did really well on that problem, but it sort of sets the stage for later. The two people I was observing both said, "Well, the $\frac{2}{3}$ ds has nothing to do with the picture. That doesn't make sense at all." And they just went to the reducing algorithm. Then, because there was extra time, they go "Oh well, it might." And they started to see that relationship between it. But their first response was "That picture just doesn't apply to the $\frac{2}{3}$ ds at all." So that was really fun.

I really liked the real deep mathematical arguments that they made to convince each other. I think the process allows things like mathematical connections, like the boy who came up with "Well, it's a fractional model." When they looked at the second example of Valerie and Cindy, and they said it's $\frac{1}{4}$ and $\frac{3}{4}$. That's a really nice mathematical connection to make. But I was even more impressed with how they were really talking about the mathematics of proportions and ratios, when they were discussing why the answers were wrong. So if the person had 8 cups of cream, or chocolate, then there would have only been 1 cup of cream, and that doesn't make sense because there's 1 cup of cream to 2. So they took those answers and did real mathematical justifications. Or when it said that the answer was 2 cups of chocolate, they go, "No, because that would mean that there were 7 cups of cream, but the cream is supposed to be smaller." So they really have that sense of proportionality in mathematical justification, when they made their answers. That was really nice to look at.

Then when they went to the next problem after doing Cindy and Valerie, one of the students drew a 3x3 box and filled it all in. Because the one for Cindy and Valerie, they were given the box. I really wanted to butt in and say, "How did you know what size you needed to make your little grid to fill in?" You know, how they figured that out. I just think that there was a real lot about mathematics of proportions that came out in their strategies. Different ways that they went about doing it. I saw one student with a nice equation of $(2 \times 3) + (3 \times 1) = 9$. Students who drew it all out. Just nice pieces of work. One of the things that happened for me was because Hillary and Carolyn said that there were a lot of mistakes in student work, but when they were having the class discussion, they were so articulate about the correct mathematics. So that I think that there's something significant that happens when they're trying to discuss an idea that's different, maybe, than their thought process that happens when they're just like working on a piece of paper. I'd like to explore that at a later time. They just seemed to approach the amount of sense-making they need to do in that class discussion differently than the amount of sense-making they do when they're just doing a piece of paper.