

ELYSHA PASSEGGI: So this morning we're going to do a number talk and we're going to talk about the same kind of ideas that we were talking about on Wednesday. So I want to just look back at what we discovered and talked about on Wednesday. Does anybody remember what we kind of realized when we were talking on Wednesday? Erik?

STUDENT: We kind of did like the problems that you had to try to see if it might be true or false without thinking and just looking at the numbers and you knew that they were true or false.

ELYSHA PASSEGGI: So was it without thinking or was it without something different?

STUDENT: Without something different.

ELYSHA PASSEGGI: What was it? Rebecca?

STUDENT: Without a calculation.

ELYSHA PASSEGGI: Without doing calculations. And how many people find that challenging? Did anybody come up with something that made it a little bit easier to not calculate the numbers? Did we talk about anything that made it easier to not calculate? Olivia?

STUDENT: Looking at it through relationships.

ELYSHA PASSEGGI: What do you mean by that?

STUDENT: Um, the numbers are each one less than thirty-two and thirty-three and forty-four and forty-three.

ELYSHA PASSEGGI: Okay, so you noticed a relationship between forty-four and forty-three, that forty-three was one less than forty-four and thirty-two was one less than thirty-three? And that was helpful?

STUDENT: Yeah.

ELYSHA PASSEGGI: Anybody has anything else to add? Christopher?

STUDENT: It's a little bit different from the fact families.

ELYSHA PASSEGGI: Okay and how did that help you?

STUDENT: Well, say that I knew like um...um say that this was just a made up problem and I knew three, four, if you multiply them, they equal twelve then (inaudible). What times four equals twelve? Did they...it's like a fact family, you would just have to answer the question.

ELYSHA PASSEGGI: Did knowing the multiplication of the fact families helped us with these problems?

STUDENT: No, kind of.

ELYSHA PASSEGGI: Not as much? Kind of?

STUDENT: Not as much but kind of.

ELYSHA PASSEGGI: Ok. Alright, so I want you to think about that today when we're solving some other ones and see if fact families help you, okay. And we will talk about that again. Okay, I want to show you a couple...we're going to do the same type of thing. I'm going to show a true or false problem and I want you to take some time. Don't put your hands up right away. I want you to think about it and think if the problem is true or false without doing calculations in your head okay. And then we'll talk about how you know. Alright, the first one is  $5 + 7 = 12$ . Just keep your hands down for a second and think about it. Is it true or false and how do you know? Okay, who thinks they know? Annabelle what did you do?

STUDENT: I split the five to three and two because seven plus three is ten and I just added the leftover two.

ELYSHA PASSEGGI: Okay, so you split this into a three and a two and you knew seven plus three equals ten. And then you added the two to get twelve. Did anybody do it in a different way? Erik?

STUDENT: Well I knew that six plus six also equals twelve and then I switched one of the digits...well I took one off the seven and put it onto the five and added them together.

ELYSHA PASSEGGI: Okay, so you traded one...you added one here like that?

STUDENT: Yeah.

ELYSHA PASSEGGI: Okay. Alright, does everyone agree that this is true? Do you disagree?

STUDENT: Okay, let's look at the next one. Now see if you could figure this out without doing any calculations.  $21 + 39 = 50$ . True or false? Daniel what do you think?

STUDENT: I think it's true because nine plus one equals ten and...wait, no.

ELYSHA PASSEGGI: Okay. So what did you just realized that made you change your mind?

STUDENT: Well because at first...because thirty plus twenty would equal fifty but that equals sixty because it takes the nine and one from the thirty-nine and the twenty-one is ten, which would add ten to it.

ELYSHA PASSEGGI: Okay, so first you started by looking at the ones and then I realized that you said "oh wait a second" and I think that what you realized is that when you looked at the twenty and thirty that you already had fifty, but there was going to be more to add to that, which would give you more than fifty. Am I correct? Okay, anybody want to add something different to this one or saw this in a different way? Heather?

STUDENT: Well, I thought it as like you just took the one from the twenty and move it to the thirty-nine, which made you have forty and twenty and that's how you sort of know the answer is not right because it's ten over fifty.

ELYSHA PASSEGGI: Anybody see it in a different way and wants to add anything else? Ellie?

STUDENT: Well I kind of did it differently because as soon as I read it, I was almost positive it was fifty then I did it again and then I got sixty. So you can't go through it really fast because then you don't get all the calculations.

ELYSHA PASSEGGI: Okay, so when you just glanced at it you thought that it was true and then when you took a second glance and you kind of started calculating, you thought maybe it wasn't true, correct? Christopher?

STUDENT: Well I thought this was false and what I did is that I rounded the numbers. So since I added one to the thirty-nine to make it forty and I subtracted one to the twenty-one to make it twenty, that basically is like doing nothing because zero plus one equals one and minus one equals zero. So zero zero is the same number and so well I thought that...

ELYSHA PASSEGGI: These zeros?

STUDENT: No, it's just like a random equation.

ELYSHA PASSEGGI: Okay, so I'm just wondering then you're saying that zero plus zero...I'm just wondering how that is connected to this. I'm just not understanding.

STUDENT: Oh. Well that's not what I mean. Like it's just like if you add one and then subtract one, it's like basically equals nothing.

ELYSHA PASSEGGI: Oh, so you're saying that because you added one here and you subtracted one from here that it kind of was cancelling each other out, or that you were doing the same thing to each number...kind of the opposite?

STUDENT: (inaudible)

ELYSHA PASSEGGI: Okay, that makes sense to me now, thank you. Anybody disagree or want to add anything else? Erick?

STUDENT: Well I knew that twenty plus thirty equals fifty, so then if you even had like a one or a nine that you already will be over. So I knew that that was false.

ELYSHA PASSEGGI: Okay, so looking at this and then knowing the one, which I think Daniel discovered when he went back to it. Okay, one more. Twenty seconds to think about it.  $62 + 76 = 238$ . True or false and why? What do you think Kate?

STUDENT: Um, I think it's false.

ELYSHA PASSEGGI: Why?

STUDENT: Because sixty-two and seventy-six will equal something in the one hundreds. Because to get two hundred you need at least one hundred and one hundred to equal two hundred, so I think it's false.

ELYSHA PASSEGGI: Okay, anybody want to add to that or saw it in a different way. Taniya?

STUDENT: Well I knew that six plus seven equals thirteen. So that would equal one hundred and thirty and then...two plus six equals eight so you may need a lot more to make it at least two hundred.

ELYSHA PASSEGGI: Okay, so kind of what Kate was saying, that it looked like it was going to be in the hundreds.

STUDENT: Yeah.

ELYSHA PASSEGGI: Okay, anybody see it differently? Rosie?

STUDENT: At first glance I thought it was true but then I wanted to just like make sure, so I added the sixty and the seventy and I got a hundred and thirty. And then like "what, that couldn't work." And then six plus two equals eight and that's actually one hundred and thirty-eight, not two hundred and thirty-eight.

ELYSHA PASSEGGI: I have a question. Looking at this problem here, when you see this number two hundred and thirty-eight, does this look like a reasonable answer for this problem?

STUDENT: No. The numbers are too small.

ELYSHA PASSEGGI: Ah. Aden what do you mean by that, the numbers are too small?

STUDENT: They have to be bigger to get to a bigger number. Those need like...I don't know. I don't think those are big numbers to get to two hundred thirty-eight. They just don't look big enough.

ELYSHA PASSEGGI: Okay, Frederick do you want to add to that?

STUDENT: Um, yeah. Well, I did two things. Um, I did two different things. So first I already knew that that was a hundred and thirty-eight, but also two two digit numbers no matter what, if it's addition cannot be more than one hundred ninety-eight.

ELYSHA PASSEGGI: So you're saying that no matter what, two two digit numbers...

STUDENT: Added together.

ELYSHA PASSEGGI: Added together can never be more than...

STUDENT: A hundred ninety-eight.

ELYSHA PASSEGGI: Interesting, does anybody agree or disagree with that? What do you think? I think that's an interesting comment. I don't know if it's true, I haven't tried on everything. Why would this be true?

STUDENT: Why wouldn't it?

ELYSHA PASSEGGI: Or why wouldn't it? Maddy?

STUDENT: I think it's true because the highest two digit number you could have is ninety-nine and ninety-nine and ninety-nine is a hundred and ninety-eight.

ELYSHA PASSEGGI: Interesting. So using that rule, if it was true and you saw two two digit numbers and then an answer that was larger than two hundred, could you assume right away that it's false?

STUDENTS: Yes.

ELYSHA PASSEGGI: If Frederick's rule is true?

STUDENT: ...exactly the last number but it's the highest.

ELYSHA PASSEGGI: The highest?

STUDENT: Two digits.

ELYSHA PASSEGGI: Two digits?

STUDENT: The highest two digit.

ELYSHA PASSEGGI: Okay. Alright, I have put a question up here that is the same question that I asked you the other day. Can you figure out in your head -- now without calculating, that means without adding this and taking from here and putting the...try and just look at the relationships of the numbers. Can you figure out in your head if these problems are true or false? Ready? Okay. Take a second to think about it. Don't put your hands up, just think about it for a second.  $32 + 28 = 33 + 27$ . Don't put your hands up yet, wait. Think about it.

STUDENT: I think it's true.

ELYSHA PASSEGGI: Why?

STUDENT: Because if you add...well I just think it's true because the three would fit into the seven on thirty-three plus twenty-seven and then the two would fit into the eight. They both equal ten so I think it's true.

ELYSHA PASSEGGI: So what did you just do to figure that out?

STUDENT: Two plus eight is ten and three plus seven...

ELYSHA PASSEGGI: But what strategy did you use to figure that out?

STUDENT: Well, calculate.

ELYSHA PASSEGGI: Doing calculations. Which you can definitely do to get the answer but I'm just wondering did anybody did it a different way without calculating?

STUDENT: Well the thirty-two and the thirty-three are one number apart.

ELYSHA PASSEGGI: Okay.

STUDENT: And then the twenty-eight and the twenty-seven.

ELYSHA PASSEGGI: Are what?

STUDENT: Are one number apart.

ELYSHA PASSEGGI: Okay, if I have from thirty-two to thirty-three, you're saying that the difference is one? So what changed from thirty-two to thirty-three?

STUDENT: It got bigger.

ELYSHA PASSEGGI: It got bigger by one? So could I record that by saying that? Does that make sense? And then what did you notice about the twenty-eight and the twenty-seven?

STUDENT: Same thing.

ELYSHA PASSEGGI: The same thing? Okay, so from twenty-eight to twenty-seven...

STUDENT: It got minus.

ELYSHA PASSEGGI: Oh, minus. Interesting. Kate what do you think?

STUDENT: I think they equal the same thing because if you put the twenty-eight on the thirty-two it would be thirty-three and it would balance both sides.

ELYSHA PASSEGGI: Okay wait, wait, say that again.

STUDENT: If you put one from the twenty-eight onto the thirty-two then the thirty-two would become thirty-three and it would be the same as the other side. And if you put one from the thirty-three onto the twenty-seven then it would become twenty-eight and both the sides would be equal.

ELYSHA PASSEGGI: Okay, so you're saying -- you're looking at the relationship of these two numbers and these two numbers, where Rose was looking at the relationship of these two numbers and these two numbers. Is that correct?

STUDENT: I was kind of looking at those relationships also. The other numbers help.

ELYSHA PASSEGGI: Okay, so you're taking one from this and adding it to that and then you're taking one from here and adding it to that. And then this one you're going this way right? So you're taking one from here and adding it to there, so that both sides would be twenty...thirty-two plus twenty-eight? Okay, Heather?

STUDENT: When Rose did it, the one sort of canceled each other out like on hands on equations, so that they basically just canceled each other out.

ELYSHA PASSEGGI: This way? Okay, so you're saying that because the change is doing the opposite thing, they're canceling one another out?

STUDENT: Yeah.

ELYSHA PASSEGGI: Okay, the same kind of ideas as hands on equations.

STUDENT: Yeah.

ELYSHA PASSEGGI: Okay, so do you then agree that this is true or false?

STUDENTS: True.

ELYSHA PASSEGGI: Okay. Alright, let's look at another one. Think about it. See if you think this is true or false. Think about it. Don't put your hands up yet. Take a second to think about the problem.  $41 + 36 = 44 + 32$ . Does anybody see anything here that would help us figure this out? Okay, so you were rounding. So then after you rounded, what did you do?

STUDENT: Well, um that's all I did.

ELYSHA PASSEGGI: Okay, so you started by rounding. Did anybody do it in a different way? Ryan?

STUDENT: Well, I looked at the relationship of the numbers.

ELYSHA PASSEGGI: Okay. And what did you notice?

STUDENT: And I noticed that from the forty-one to the forty-four you added three and from the thirty-two to the thirty-six you added four.

ELYSHA PASSEGGI: Thirty-two to thirty-six you added four. Okay.

STUDENT: They told me that one side must be bigger than another because the numbers... because they added more on one side than another.

ELYSHA PASSEGGI: Okay, so you're saying that by figuring out what was changing and noticing that this was a bigger number or bigger change that one side would be bigger than the other? Interesting. Does anybody have anything to add? Olivia?

STUDENT: Well that side being plus three and the other side being plus four, um I think it would mean that it would be false because they're two different numbers, so you can't cancel them out.

ELYSHA PASSEGGI: So since these are two different numbers, you can't just say we did the same thing on both sides? So you can't cancel that out? Did anybody see it in a different way besides plus three and plus four? Ellie what did you see?

STUDENT: I kind of did the same thing as Ryan but I know how to decide which one is bigger, which side is bigger. You could just add the one because you know that the forty and the thirty are going to be the same on each side, so whichever ones are higher, that's the higher.

ELYSHA PASSEGGI: That would definitely work but that is calculating. Did anybody see this differently though? I'm just curious, besides plus three and plus four, did anybody see it in a different way?

STUDENT: I did it like plus three and like minus four because thirty-six to thirty two is minus four.

ELYSHA PASSEGGI: So you saw it since you were changing from forty-one...what was changing from forty-one to forty-four and what was changing from thirty-six to thirty-two?

STUDENT: Yeah.

ELYSHA PASSEGGI: I'm just going to draw that underneath here. And what did you see changing?

STUDENT: Minus four.

ELYSHA PASSEGGI: Minus four. Did that change your opinion? Did you still see it as being false or?

STUDENT: Yes.

ELYSHA PASSEGGI: You did. And why did you see it as being false?

STUDENT: Because adding three and subtracting four, it doesn't (inaudible)

ELYSHA PASSEGGI: Okay, so the change is bigger here than here?

STUDENT: Yeah.

ELYSHA PASSEGGI: So does everyone agree that this one is false?

STUDENT: Yes.

ELYSHA PASSEGGI: Okay. Alright, one more.  $29 + 56 = 27 + 58$ . Think about it for a second. Alright, does anybody have something? Maddy what do you see?

STUDENT: I think it's true because wouldn't you know that the tens are going to be the same? And if you add the nine and the six...well if you take one from the nine and added to the six then you get seven and eight. And that way you would know it's the same.

ELYSHA PASSEGGI: So this kind of strategy. If you trade one from here...if you trade just one... Is that what you're saying? Or trade something different than one?

STUDENT: If you trade, wait. If you do minus one from the twenty-nine and then plus one to the fifty-six then you get twenty-seven and fifty-eight, which is the same as the other side.



ELYSHA PASSEGGI: Okay, so twenty-nine take away one is giving you twenty-seven?

STUDENT: No, but if you take the nine and traded it, minus one, wait. If you take one from the nine and then you add the one to the six, you will get seven on the six and you'll get eight on the other one. Oh wait, never mind.

ELYSHA PASSEGGI: So if you were going to do that, what would you need to take from this twenty-nine to have it be the same on both sides? Two. And if you took two from here, you would get?

STUDENT: Fifty-eight.

ELYSHA PASSEGGI: Fifty-eight. So then it would work. So two would work. Did anybody look at this without doing calculations and just looking at the relationships of the numbers? David?

STUDENT: Well I saw the twenty-nine and fifty-eight. Well I just thought that it was true. I added twenty-nine and fifty-six and fifty-seven, that if you added, um... I just thought that it was true. I just guessed it.

ELYSHA PASSEGGI: Okay. Alright, Taniya?

STUDENT: Well I know that the twenty-nine and the fifty-eight were (inaudible).

ELYSHA PASSEGGI: You knew the twenty-nine and fifty-eight were what?

STUDENT: Well I just looked at the ones and I noticed that the eight was...well you had to add one to eight to get nine. And I just looked at the fifty-eight and twenty-nine.

ELYSHA PASSEGGI: So you noticed a relationship between these two numbers?

STUDENT: Not the tens but the ones, yeah.

ELYSHA PASSEGGI: Between these two numbers, the nine and the eight?

STUDENT: Yeah.

ELYSHA PASSEGGI: Okay. Did anybody notice a relationship between the numbers in a different way? I'm just curious because I'm hearing a lot of calculating, which will also give you the right answer. I'm just wondering if anybody noticed a relationship between these numbers. Rebecca?

STUDENT: I knew that twenty-nine was two more than twenty-seven.

ELYSHA PASSEGGI: Okay.

STUDENT: And fifty-six was two less than fifty-eight.

ELYSHA PASSEGGI: Okay. So, I'm just going to write this in a different way. Tell me if you think this makes sense. So you're saying that you had fifty-eight take away two equals fifty-six?

STUDENT: Yeah.

ELYSHA PASSEGGI: And that you had twenty-seven, sorry, twenty-seven plus two equals twenty-nine. So what would both the sides be equal to here? Is this true or false? Anybody know? Kate?

STUDENT: Since that's plus two and that's minus two then the other side would be plus two and minus two.

ELYSHA PASSEGGI: Okay, so you're saying because one is going up by two and one is going down by two then it's going to equal the sides because you're changing them in the opposite way? Which is what Heather was saying here, where it's canceling them out based on what we learned about hands on equations. Aden?

STUDENT: I noticed that the two numbers on the right side is fifty-eight and twenty-seven and then the answer is the other side, fifty-six and twenty-nine. I think they're true.

ELYSHA PASSEGGI: Let's try a challenge problem. Let's see if we can do a problem where I don't give you all the numbers and let's see if you can figure out what number is missing. You want to try that? Alright, what number would you put in this box to make the number sentence true? And remember no calculating. And I want you to think about what we talked about here. Frederick what do you think?

STUDENT: I think I would put a seven in the box because four is one less than five.

ELYSHA PASSEGGI: Okay.

STUDENT: Um, and eight is one more than seven. So they cancel each other out because they come with (inaudible).

ELYSHA PASSEGGI: Interesting. Did anybody else see it that way? Anybody disagree? Let's try another one. Twelve plus seven equals eight plus what ( $12 + 7 = 8 + \underline{\quad}$ )? What do you think Rose?

STUDENT: Eleven.

ELYSHA PASSEGGI: Eleven? Derrick how did you come up with eleven?

STUDENT: Because eight is one more than seven.

ELYSHA PASSEGGI: Okay.

STUDENT: And then eleven is one less than twelve.

ELYSHA PASSEGGI: Ah. Is anybody noticing something? Is anybody noticing anything? Hannah what are you noticing?

STUDENT: That they're either going up or down one.

ELYSHA PASSEGGI: That they're either going up or down one? And that is the case for both of these? Does anybody notice anything else? Erick?

STUDENT: I notice that every time there's one arrow pointing to one side that it's always plus something. Then there's also another arrow pointing the other side that always says minus something.

ELYSHA PASSEGGI: Ellie?

STUDENT: Well I have an observation.

ELYSHA PASSEGGI: Uh-ha.

STUDENT: You did eleven going this way is minus one, it should be plus one.

ELYSHA PASSEGGI: Sorry you're right.

STUDENT: Yeah, and then also all the problems we've done are like something plus something equals something plus something.

ELYSHA PASSEGGI: Okay, past two, it's a number sentence (inaudible). Okay, Christopher?

STUDENT: Well each time we've done like one of those, well the first, um, seven is one less than eight and eight is one more than seven. So automatically I know that the twelve -- the one in the box is going to be one less than that.

ELYSHA PASSEGGI: Okay, interesting. Let's try the next one then and see if any of those theories work. Fifty-three plus what equals forty-nine plus thirty-five ( $53 + \underline{\quad} = 49 + 35$ )? What do you think? Does anybody have an idea on this one? I want you to take a second and talk to your shoulder partner and see what they think about this one.

STUDENT: I would think that since I made an observation for that there is one number on either side... If there's one number on one side and one number on the other side and this number is bigger, then this number has to be bigger. So I think that strategy...

ELYSHA PASSEGGI: Alright does anybody have an idea of what number is missing there? Anybody have an idea of what number is missing? Maddy what do you think?

STUDENT: Thirty-one.

ELYSHA PASSEGGI: Thirty-one? How did you guys come up with thirty-one? Oh here? So forty plus thirty is seventy?

STUDENT: Yeah. And nine and five is fourteen.

ELYSHA PASSEGGI: So you figured it out by doing calculations to figure it out?

STUDENT: Oh, I know.

ELYSHA PASSEGGI: You disagree?

STUDENT: Yes.

ELYSHA PASSEGGI: Okay, what did you do?

STUDENT: I did forty-nine is four less than fifty-three and so if it's four less, then you need four more for the thirty-one. The thirty-one is four more than the thirty-five and that's how I figured it out.

ELYSHA PASSEGGI: So you had to figure out what number plus four is going to give you thirty-five? Anybody else see it that way? Who saw it Maddy's way? One last problem. Eighteen plus twenty-four equals twenty-one plus what ( $18 + 24 = 21 + \underline{\quad}$ )? Alright, what did you come up with? Quinn what did you come up with?

STUDENT: Twenty-one.

ELYSHA PASSEGGI: And how did you figure out twenty-one?

STUDENT: Well I took off three from the twenty-four and added it to the eighteen.

ELYSHA PASSEGGI: Oh, you did this. So that's what you did?

STUDENT: And then I have twenty-one and twenty-one.

ELYSHA PASSEGGI: How did you know this was twenty-one though?

STUDENT: Because three plus eighteen is twenty-one and twenty-four minus three is twenty-one.

ELYSHA PASSEGGI: Oh, so you wanted to get a twenty-one and a twenty-one on both sides that you could just see it. Did anybody see it in a different way, Olivia?

STUDENT: So first, I looked at the twenty-four and twenty-one.

ELYSHA PASSEGGI: These two?

STUDENT: Yeah. And it was minus three.

ELYSHA PASSEGGI: Okay, let me just cross that out.

STUDENT: And so from all the other problems it was...I was going to see if it was true or false. I made it plus three on the eighteen, plus three and that, like...it was equal sort of but...I don't really know exactly how to explain it.

ELYSHA PASSEGGI: It was equal sort of? Are you meaning the threes were the same?

STUDENT: Yeah. And then I looked at the eighteen and twenty-four and I kind of did what Quinn did to make it twenty-one and twenty-one.

ELYSHA PASSEGGI: I think that's an interesting observation. Alright, you guys did a really great job on these problems.