Interpreting Fractions

Silicon Valley Mathematics Initiative's

Formative Re-Engaging Lesson

Fifth Grade

Interpreting Fractions

Mathematical goals

This lesson unit is intended to help you assess how well students are able to translate between the symbolic notation for fractions and representations of fractional area, measurement and sets, and to identify and help students who have difficulty:

constructing fractional parts of a whole;
distinguishing between the part, the whole, and the fraction that names the part/whole relationship;
translating between representations;
understanding the meaning of the words numerator and denominator.

Standards addressed

This lesson relates to the following Common Core State Standards:

- Third Grade Geometry 2: Partition shapes into parts with equal areas.
- Third Grade Number and Operations Fractions: Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and portioning it into b equal parts.
- Fourth Grade Number and Operations: Fractions 1. Explain why a fraction *a/b* is equivalent to a fraction (n x *a*)(n x *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

This lesson relates to the Mathematical Practice:

- Modeling: Mathematically proficient students can apply the mathematics they know to solve problems. This might mean at early grades being able to write an equation or problem to demonstrate a situation.

Introduction

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understanding and difficulties.
- Students work in pairs or threes on collaborative discussion tasks. As they do this, they translate between fractions represented by a number and different representations for the given fraction.
- Students take the individual assessment again, compare their second attempt to their first, and reflect upon what they have learned from this lesson and what they are still struggling to understand.

Materials required

Each student will need one copy of the individual assessment task.
Each pair of learners will need the following:

- Card Set A – *Fractions Represented by a Number*; Card Set B – *Area Models*; Card Set C – *Measurement Models*; Card Set D – *Set Models*: Card Set E - *Fractional Situations*.

Please note:
It is helpful for sorting and maintaining control over the many different cards to have each card set copied in a different color.

- Pattern blocks, fraction pie pieces, geoboards and rubber bands, grid paper, fraction strips, fraction bars or cuisenaire rods, two-colored counters, and blank paper should be accessible to students.
- A glue stick, a felt tip pen, a large sheet of construction paper or butcher paper for making a poster are needed. The poster material should be available for the first pairing of Card Set A and Card Set B. If students make changes as the different card sets are added, extra cards should be available to them to replace changed matchings.

  Please Note:
  If cards are laminated and tape is used, the cards may easily be re-arranged as students determine equivalent representations.

- Envelopes and paper clips for storing matched cards in between lessons if time is short.

Resources

Heibert, James 1997 **Making Sense,** Portsmouth, New Hampshire: Heinemann Press ISBN 0-435-07132-7

How to Teach Math as a Social Activity Edutopia.org video on building community norms around math discussions.

Time needed

The lesson will need at least two one-hour sessions. Timings given are only approximate. Exact timings will depend on the needs of the class.

Before the lesson

Individual Assessment Task: *Fractions*

The assessment task [page 17 in this unit]. , *Fractions* (2005 MARS Grade 5), should be completed before the lesson. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach a lesson using a similar task which should help them.

It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started then ask questions that help them understand what is required, but don't do it for them!

Possible Questions to Pose to Students Who May Be Struggling:
"What is the question asking you to do?"
"What do you know about the number line that may help you?"

Assessing students' responses

Collect student responses to the task and make some notes on what their work reveals about their current levels of understanding. Look for similarities among the papers regarding student understanding and difficulties. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself so that you may prepare carefully. Do not grade students' work at this stage. Research shows that this will be counterproductive as it will encourage students to compare their grades and distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them.

As students work on the lesson, consider using the suggested questions and prompts listed below that arose from student work on the pre-assessment task *Fractions* in the trialing process.

COMMON ISSUES	SUGGESTED QUESTIONS and PROMPTS
Student interprets the quantity value of a fraction by a part [the whole number in the numerator or the denominator] rather than the value of the fraction as a whole.	• What does the fraction ½ mean? Draw me a picture or tell me a story about ½. In this picture or story, show me where or what the 1 is. In this picture story, show me where or what the 2 is. • What does the fraction ¼ mean? Draw me a picture or tell me a story. In this picture or story, show me where and what the 1 is. In this picture or story, show me where or what the 4 is. • Which fraction is bigger, ½ or ¼? How do you know? • What are you comparing? • Describe and support your answer with your drawings or stories. • On a number line labeled 0 to 1, where would you place the ½? The ¼? Why?

Student correctly places the two fractions on the proper side of ½ on the number line, but there is no justification.	• Tell me your reason for placing the fraction 2/3 greater than ½ on the number line. • Can you draw a picture or tell a story to help you? What would it be? • Can you put additional markings on the number line to help you share your thinking out loud? Where would you put them and why? • Tell me your reason for placing the fraction 2/5 to the left of ½ on the number line. • Can you draw a picture or tell a story to help you explain your reason? What would it be? • Can you put additional markings on the number line to help you share your thinking out loud? Where would you put them and why?
Student correctly places the two fractions on the proper side of ½ on the number line and gives a mathematical reason for the placement, usually finding common denominators or converting to decimals or percents. However, student struggles with comparing the size of 2/3 and 2/5 to the ½ and placing them accurately on the number line according to their value or quantity.	• Tell me more about how you placed 2/5 on the number line. • How did you determine the distance from ½? • Can this reasoning help you place 2/3 on the number line? How? • What do you know about the distance between ½ and 2/3? • What do you know about the distance between 2/3 and __? • Are there other mathematical representations for fractions that might help you justify which fraction is closer to ½?
The student successfully completes and justifies the performance task *Fractions.*	• Where on the number line would you place the fractions 3/7 and 5/9? • Why would you place them there? • Show me at least 3 different mathematical ways to justify your placement on the number line.

Suggested lesson outline

Class introduction

[15 minutes]

Hold a short question and answer session, using mini whiteboards. Ask students to share and justify their solutions. This is an opportunity to surface student misconceptions and to discuss possible varied responses.

"On your mini whiteboards, show me a rectangle. Show me this rectangle divided into two halves."
"Show me a circle divided into four fourths. Show me three-fourths of this circle (write ¾)."

"On your mini whiteboards, show me a line having a dot at its beginning and a dot at its end. Show me this line segment divided into four-fourths. Show me this same line segment divided into eight-eighths. Show me on this line segment where to find seven-eighths (write 7/8)."

"On your mini whiteboards, show me a set of six buttons using circles for the buttons. Show me one-half of this set of buttons (write 1/2). What does this fraction represent?"
Have students justify their numerical representation.

Please note:
3/6 or ½ is acceptable.

"Show me a set of 5 tiles using a square to represent each tile. Show me three-fifths of this set of 5 tiles (write 3/5). What fraction does this represent?"

Collaborative task: Matching Card Set A (*Fractions Represented by a Number)* with Card Set B (*Area Models)*
[15 minutes]
The first task is designed to help students interpret the quantity of fractions represented by a number and realize that these fractions define two things: the area of each equal-sized piece of the whole and the number of equal-sized pieces. Organize students into groups of two or three and give out Card Set A (*Fractions Represented by a Number*) and Card Set B (*Area Models)*. Students will cut the cards only when the pair has agreed to a match.

Please note:
Students are determining the fractional part of the shaded area.

"Your task is to match the fractions represented by a number with the corresponding area model. Take turns with your partner to choose a fraction and the area model which represents that quantity. When you and your partner are in agreement with each other, then and only then may you cut out the two cards that match. Place these side by side on the table and explain how you know that they match. If you cannot find a matching card, write your own, using the blank cards."

Please note:
Students will need to make area cards and fractions represented by a number to match.

When a pair of students is satisfied with their matchings for Card Sets A and B, have the pair share their matchings with another pair of students from a different part of the room. (Recording the matches on a whiteboard eliminates the need to carry loose cards on a poster over to another pair of students.) If the two pairs are in disagreement, a discussion around the mathematics should ensue. When all the matchings have been discussed, each pair has the opportunity to make changes to their original matchings. Only when a pair feels mathematically

comfortable with their matchings may Card Sets A and B be glued on the poster. Now is the time for students to get a large piece of construction paper or butcher paper and a glue stick to begin making their poster.

Please Note:
The gluing is taking place in increments only because there are a large number of cards to organize. Again, have extra cards ready for students who may change their minds as they do more card sets.

Likewise, this interim processing may be done at the teacher's discretion after each addition of another card set.

Some students may notice that a few of the fractions represented by a number are equivalent. You do not need to comment on this at this stage. This is a noticing that warrants discussion with the whole group.

Throughout this collaborative activity, students will be asked to occasionally create representations for area, measurement or sets. Drawings provide opportunities for you to assess students' understanding. It is not until a student is asked to draw a fractional representation that it becomes apparent that the notion of equal parts may not have been incorporated into his or her emergent understanding of fractions. It is also important to distinguish between incorrect drawings of fractions due to drawing skills and those that are the result of mistaken ideas.

Collaborative task: Matching Card Set A (*Fractions Represented by a Number)* and Card Set B (*Area Models)* with Card Set C (*Measurement Models)*
[15 minutes]
Give out Card Set C and ask students to match these to the card pairs already on the table. Some models are missing. Students will need to draw their own versions. With measurement models, length is compared instead of area. Having manipulatives available provides more opportunity for trial and error, exploration and discussion. Encourage students to justify how and why a particular measurement model represents the same fractional part of a corresponding area model.

Collaborative task: Matching Card Set A (*Fractions Represented by a Number)*, Card Set B (*Area Models)* and Card Set C (*Measurement Models)* with Card Set D (*Set Models)*
[15 minutes]
Set Models help to connect fractions represented by a number to many real-world uses. Give each group of students Card Set D and ask students to match these to the triple-card sets already on the table. Some models are missing. Students will need to draw their own versions. Again, encourage students to justify how and why a particular set model represents the same fractional part as the previous set of cards.

Collaborative task: Matching *Card Set A (Fractions Represented by a Number)*, Card Set B (*Area Models)* and Card Set C (*Measurement Models)* and Card Set D (*Set Models)* with Card Set E (*Fractional Situation Cards)*
[15 minutes]
The situational models help to connect fractions represented by a number to our real world of fractions. Give each group of students Card Set E.

"Each of the cards in Set E is a fraction situation or story. I want you to match these situation cards to your groupings already on the table. Remember, you may have to write your own situation if you can't find a matching card. When you reach agreement with your partner, paste down your final arrangement onto the large construction paper, creating a poster. Your final arrangement will connect four different models to each fraction represented by a number."

These posters will be displayed for the final class discussion. As students do the matching and pasting or gluing, go around and encourage students to explain why particular cards go together.

Lesson Extension
When a pair feels that they have matched all the card sets and can mathematically justify each group, students may cut the poster into strips with each matching group on one strip. These strips may then be organized from least to greatest or greatest to least and glued to another poster in the preferred order as directed by the teacher.

Plenary Discussion [15 minutes]
Hold a whole class interactive discussion to review what has been learned over this lesson. Ask each pair of students to justify one of their groupings and explain to the whole class why the different types of models convey the same fractional quantity.

Then use mini whiteboards and questioning to begin to assess students' learning. For each representation have students write the corresponding fraction.

"Show me three-fourths of a whole rectangle. Now show me three-fourths of a set of sticks."

"Show me two-thirds of a folded paper strip. Now show me two-thirds of a set of hearts."

"Show me one and one-quarter pizzas. Now show me one and one-quarter on a number line."

Individual work (30 minutes)
Finally, give students a new copy of the task *Fractions* (2005 MARS Grade 5) [page 17 in this lesson]. Ask students to have another go at it. When students have completed the second attempt, pass out their original work to them and ask them to write a reflective paragraph about what they have learned from the lesson that helped them on the task and what they are still struggling to understand.

Solutions

This table is for your convenience only.
It is helpful not to refer to cards by these letters in class, but rather to the content of the cards.

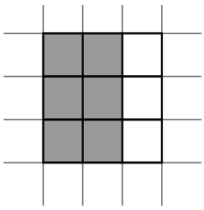
Fraction symbol	Area Model	Measurement Model	Set Model	Fraction Situations
A1	**B8**		**D7**	**E3**
A2	**B10**	**C1**		**E6**
A4	**B3**	**C2**	**D5**	
A5	**B1**		**D3**	**E10**
A6	**B2**	**C8**		**E5**
A8	**B9**		**D8**	**E2**
A9	**B7**	**C4**		**E8**
A10		**C5**	**D4**	**E4**
A11	**B4**	**C7**	**D1**	**E7**
A12	**B6**	**C6**	**D6**	

CARD SET A

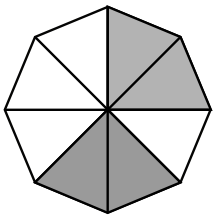
$\frac{1}{4}$ A1	$\frac{5}{6}$ A2
A3	$\frac{3}{10}$ A4
$\frac{6}{9}$ A5	$\frac{1}{2}$ A6
A7	$\frac{9}{5}$ A8
$\frac{10}{12}$ A9	$\frac{5}{8}$ A10

A11	A12

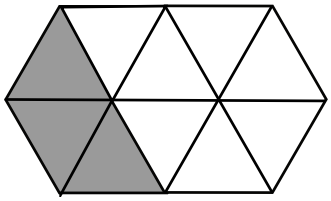
CARD SET B



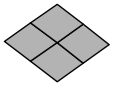
B1

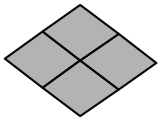


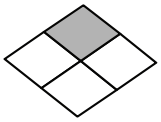
B2



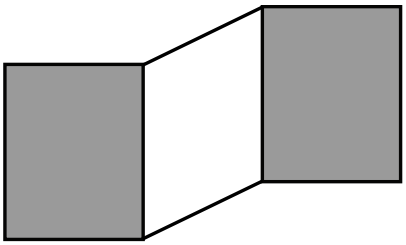
B3

If  is one whole:

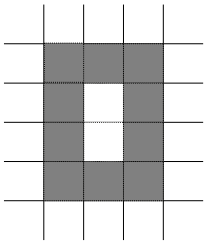




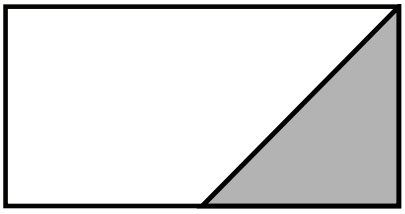
B4



B6

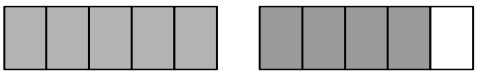


B7

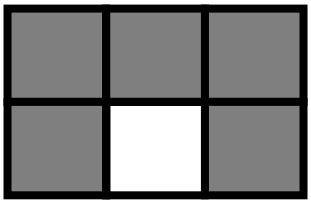


B8

If  is one whole:



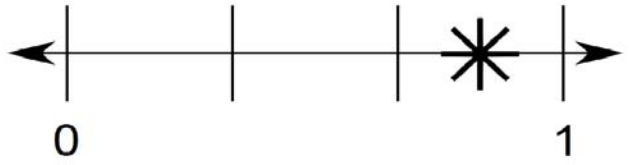
B9



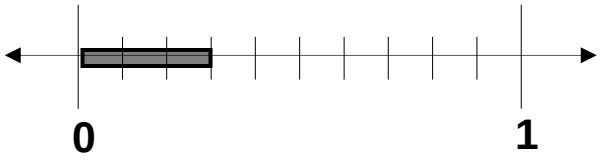
B10

B11	B12

CARD SET C



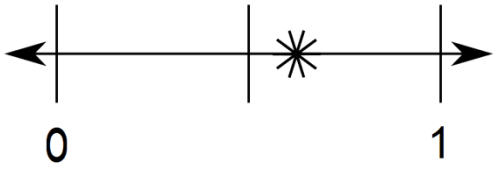
C1



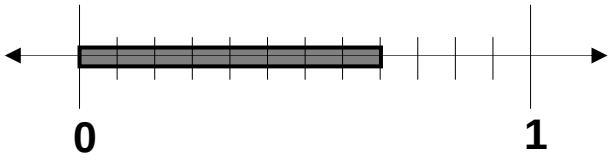
C2

C3

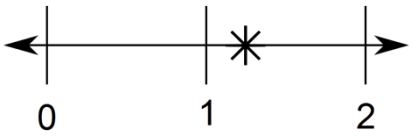
C4



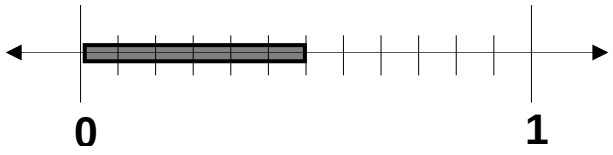
C5



C6



C7

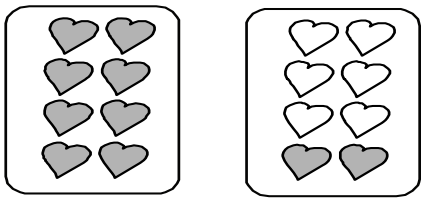


C8

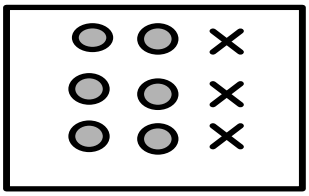
C9

C10

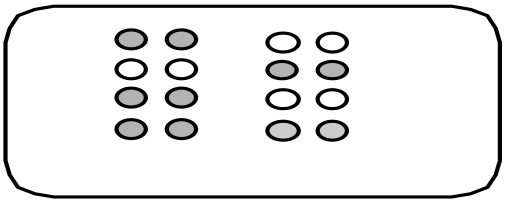
CARD SET D



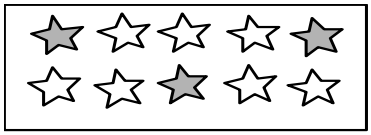
D1



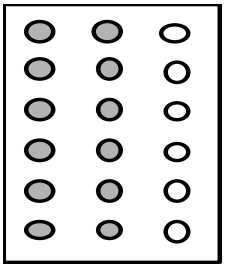
D3



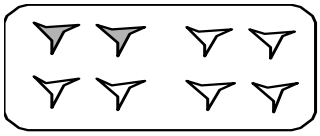
D4



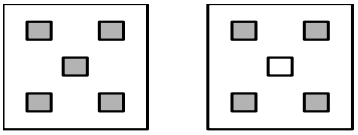
D5



D6



D7



D8

D9

D10

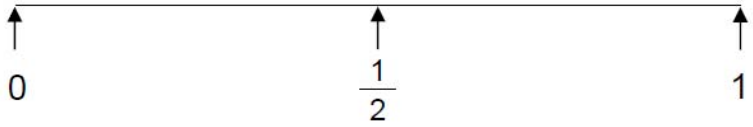
CARD SET E

	Melinda and Jamie ordered two pizzas. Melinda ate all but 1/5 of a pepperoni pizza and Jamie ate all of the cheese pizza. What fraction of all the pizzas did they eat together? E2
We cut the watermelon into 4 pieces. 3 pieces of the watermelon were eaten. What fraction of the watermelon is left? E3	Mom bought 8 apples. Marsha ate three, Janice ate 2 and Jacob ate the other three. What fraction of the apples did Marsha and Janice eat? E4
For exercise, Jody will dance for 30 minutes. What fraction of her exercise will it be after 15 minutes? E5	Mario bought new socks. The package had 6 pairs: 3 white pairs and 3 black pairs. He lost one of the black pairs. What fraction of the new socks did he still have? E6
The Gomez family bought 2 pies for Thanksgiving dinner. They cut each pie into 8 slices. That night the family ate all of one pie and 2 slices of the 2nd pie. What fraction of all of the two pies did the Gomez family eat? E7	Ben is making friendship bracelets. He usually strings 12 beads to make a bracelet. He was 2 beads short on his last bracelet. What fraction of the last bracelet was finished? E8
E9	Ms. Kennedy put 9 tiles in a bag. Some were yellow and some were red. The students found out that there were 3 red tiles and the rest were yellow. What fraction of all the tiles in the bag were yellow? E10
E11	E12

Fractions

This problem gives you the chance to:
- show the position of fractions on a number line
- compare the sizes of fractions

Here is a number line.



1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? _____

   Explain how you figured it out.