

Interpreting **Multiplication and Division**

**Silicon Valley Mathematics
Initiative's**

Formative Re-Engaging Lesson

Third Grade

Interpreting *Multiplication and Division*

Mathematical Goals

This lesson unit is intended to help you assess how well students understand the relationship between multiplication and division and how well students are able to translate between the symbolic notation for multiplication and division and different representations of: contextual word problems, words, equations/number sentences, number lines, and arrays to identify and help students who have difficulty:

- recognizing multiplicative relationships and transitioning from additive to multiplicative solutions strategies
- understanding and using the language of ‘equal groups of’ and ‘equal parts of’ to make sense of multiplication and division
- translating between different representations
- understanding the inverse relationship between multiplication and division
- understanding the meaning of the words factor, multiple, product, and quotient

Standards Addressed

This lesson relates to the following Common Core State Standards

- Third Grade Operations and Algebraic Thinking: Represent and solve problems involving multiplication and division
- Third Grade Operations and Algebraic Thinking: Understand properties of multiplication and the relationship between multiplication and division
- Third Grade Operations and Algebraic Thinking: Multiply and divide within 100
- Third Grade Operations and Algebraic Thinking: Solve problems involving the four operations and identify and explain patterns in arithmetic

This lesson relates to the Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

Introduction

This lesson unit is structured in the following way:

- Students work on their own, completing an assessment task that is designed to reveal their current understandings and difficulties.
- Students experience a Math Talk/Number Talk and introduction mini-lesson.
- Students work individually, in pairs or threes on collaborative discussion tasks. As they do this, they translate between the symbols and different representations of multiplication and division.
- Students present their findings to their classmates and participate in whole group discussions.
- Students take the individual assessment again, compare their second attempt to their first, and reflect upon what they have learned from this lesson and what they are still struggling to understand.

Materials Required

Copies of the assessment task.
Copies of Card Set A, *Contextual Word Problems*
Copies of Card Set B, *Explanations in Words*.
Copies of Card Set C, *Number Sentences/Equations*
Copies of Card Set D, *Number Lines/Measurement Models*.

Linking cubes, unifix cubes, counters, colored tiles, grid paper, and blank paper should be accessible to students as well as glue sticks, felt tip pens, and large sheets of construction paper or butcher paper for making a poster of either individual student work and/or pair work.

Please Note:

It is helpful for sorting and maintaining control over the many different cards to have each Card Set copied in a different color. Additional supplies such as overhead transparencies, document reader, and additional chart paper or poster paper may be needed, depending on procedures used for sharing and debriefing student work..

Time Needed

This lesson will need a minimum of two one-hour sessions. Exact timings will depend on the needs of the class.

Resources

Heibert, James 1997 **Making Sense**, Portsmouth, New Hampshire: Heinemann Press
ISBN 0-435-07132-7

Carpenter, Thomas 1999 **Children’s Mathematics: Cognitively Guided Instruction**, Heinemann Press ISBN 0-325-00137-5

“How to Teach Math as a Social Activity” Edutopia.org video on Building Community Norms around Math Discussions

<http://www.insidemathematics.org/index.php/classroom-video-visits>

This website will provide you with video clips of different teachers doing Math Talks/Number Talks with their students. Watching these may be helpful for those teachers who are unfamiliar with Math Talks/Number Talks. The introduction to this lesson begins with a Math Talk/Number Talk.

Before the Lesson

Individual Assessment Task: *Baking Cookies*

The assessment task, *Baking Cookies*, should be completed before the lesson [page 14]. Ask students to attempt the task on their own. Explain that they should not worry too much if they cannot understand or do everything, because you plan to teach them a lesson which should help them to understand the mathematics better.

It is important that students are allowed to answer the questions without assistance, as far as possible. If students struggle to get started, ask questions that help them understand what is required, but don't do it for them!

Possible Questions to Pose to Students Who May be Struggling

“What is the question asking you to do?”

“What do you know that will help you?”

Assessing Student Responses

Collect student responses to the task and make some notes on what their work reveals about their current levels of understanding. Look for similarities among the papers regarding student understanding and difficulties. The purpose of doing this is to forewarn you of the difficulties students may experience during the lesson itself so that you may prepare carefully. Do not grade students' work at this stage. Research shows that this will be counterproductive, as it will encourage students to compare their grades and distract their attention from the mathematics. Instead, try to understand their reasoning and think of ways in which you can help them. If students haven't had the opportunity to make sense of multiplication and division through multiple representations and/or make connections between the mathematics and real world contexts, the pre-assessment task could be frustrating, so reassure them that this assessment is to help you understand their needs and prepare the next lessons.

As students work on this lesson, consider using the suggested questions and prompts listed below that address issues identified from student work during the trialing process of the pre-assessment task *Baking Cookies*.

COMMON ISSUES	SUGGESTED QUESTIONS and PROMPTS
Student is unable to access Question #1.	<ul style="list-style-type: none"> • Tell me what you know about this problem. • If there were just one friend getting cookies, how many cookies would this friend get? • If there were two friends getting cookies, how many cookies would Jill need to bake? • How do you know this is correct? Show me. • If there were three friends getting cookies, how many cookies would Jill need to bake? • How do you know this is correct? Show me. • If there were four friends getting cookies, how many cookies would Jill need to bake? • How do you know this is correct? Show me.
Student is unable to access Question #2.	<ul style="list-style-type: none"> • Tell me what you know about this problem. • Can you draw a picture of what

	<p>you know? Show me.</p> <ul style="list-style-type: none"> • What do you need to find out? • Does your picture help you to find this out? Tell me. • How would you write this in numbers? Show me.
Student is able to access Question #1 and/or Question #2 using only one representation.	<ul style="list-style-type: none"> • Tell me what your work means. • Is there another way to represent what you are thinking? • What other ways have you seen repeated addition or multiplication represented? Show me. • When you look at these representations, can you tell me how they answer the questions? Please describe it to me.
Student struggles with Question #3.	<ul style="list-style-type: none"> • Tell me what you know and understand about this problem. • Can you tell me what the given information tells you? What does this information mean? • Can you take what you understand from the example and use that representation somewhere else? • What does this representation tell you? • What do you know in this representation? • What are you missing in this representation?

Suggested Lesson Outline

Class introduction: Math Talk/Number Talk and Mini-Lesson

[15-25 minutes]

Math Talk/Number Talk: Write 5×14 on the white board or document camera and ask students, “What is the answer to this problem? You are to think silently and give me a thumbs up when you are ready to share your thinking.” After the individual think time, ask, “Who has an idea what the solution may be?” Collect the responses and write them down for all to see. Then ask, “Who is willing to share and explain how they got their product? Which answer are you justifying?” The teacher is to record the student’s response on the white board/document camera and ask probing questions such as: “Can you tell me more about this? What were you thinking?” And then pose this question to the entire class, “Are there any questions of Student A you would like to ask?” Then ask students, “Who has used a different strategy and is willing to share?” Your job is to help facilitate the discussion on the mathematical reasoning of each student. For each student strategy, the teacher is to help students identify the key mathematical idea(s) around that particular strategy, be it ‘using friendly numbers’, ‘using numbers facts that we know’, ‘decomposing and recomposing numbers [taking apart and putting back together]’, ‘using a particular property of arithmetic such as the commutative property’, etc.

Mini-Lesson: After different strategies have been shared, reasoning articulated and clarified, with the lesson in mind, ask students to use their mini-whiteboards to record particular ‘counting strategies’ as multiplication problems. Have a discussion in class for each particular strategy so that the efficiency of multiplication can be surfaced and addressed. This helps to move students from additive repeated groupings to either the words of multiplication, ‘equal groups of’ or the symbolic notation with the multiplication symbol that represents the words ‘equal groups of’. This introduction is an opportunity to surface student misconceptions and to discuss possible varied responses.

On your mini-white boards, write what Student X’s strategy would be using words or the symbols of multiplication rather than skip counting.

LESSON EXTENSION POSSIBILITIES

There is a one to one correspondence between sets. The blank cards are an integral part of the sets because some students may not see this correspondence and will choose to use the blank cards to complete their matches. This provides invaluable information to you, the teacher, about student understanding and misconceptions.

Furthermore, to increase the cognitive demands of these collaborative tasks, selected cards of your choice may be removed from one or more sets of representations which forces students to fill in the missing representation[s] on their own and gives deeper insight into student thinking and understanding.

Whether you use the complete sets or remove some of the cards, letting students know that there are blank cards available is important because it will give additional information about students’ understandings and difficulties.

Please Note:

If one doesn’t have mini-whiteboards, one can put blank white copy paper inside of plastic sheet protectors and students may use washable markers and baby wipes to write and clean their faux mini-whiteboards.

The Lesson: Two Different Beginning Approaches (Individual or Collaborative)

ONE APPROACH

Individual Task: Matching ONE Math Story with ONE Explanation in Words

[15-20 minutes]

Materials needed: Card Set A, Card Set B.

This first task is designed to help all students grapple with making sense of just one math story/contextual word problem involving multiplication or division and the words that can be used to explain what is happening mathematically. The teacher will select a card from Card Set A *Math Stories/Contextual Word Problems* to be given to each student; there will be multiple students in the class doing the same math story. This is a purposeful design not only to promote mathematical discourse and justification in the classroom, but also to allow the teacher to differentiate for the various student capabilities in the classroom.

“Your task is to read your math story, make sense of your word problem/math story, and to solve your math story. Please use your math manipulatives to help you.”

Whole group processing: The teacher needs to have multiple copies of Card Set B *Explanations in Words* and have them posted with blue painter’s tape on poster paper or chalkboard or large white board in the front of the room.

“Let’s have everyone come to the carpet. There are other people who have the same math problem as you do. Some of you have the same answers and some of you have different answers. We need to justify what is right and correct. Using pictures, words, number sentences/equations will help us justify our solution. Remember what we did this morning in our Math Talk/NumberTalk; we justified our counting with multiplication. Now, you have the opportunity to select one of these orange cards that will describe what is happening in your math problem. Please notice that some are blank. If you cannot find a description that matches your thinking, you may take a blank one and write your own description in words.”

Have students come up in small groups to read the cards. Do not let them take any cards until all students have read all the slips and found 'their' match. Once all students have read the cards and selected the one they want, have students come up in groups not larger than 4 to take their matching card from the poster/whiteboard

Once all students have their matching card, ask them to return to their seats.

"You are to find the other people who have your math story and see if you agree or disagree with their Explanations in Words card. You must justify that your orange card is the right one. You must all come to consensus because you will be presenting your findings to the class."

After the presentations have students work in pairs or threes to match all of the cards from Card Set A with the cards from Card Set B.

"Your task is to match the math story/contextual word problem with the corresponding words. Take turns with your partner to match that story with a word card. When you and your partner are in agreement with each other, then, and only then, may you cut out the two cards that match. If you cannot find a matching card, write your own using one of the blank cards."

OR

A DIFFERENT APPROACH

Collaborative Task: Matching ONE Math Story with ONE Explanation in Words [15-20 minutes]

Materials needed: Card Set A, Card Set B.

The first task is designed to help students interpret the mathematics from a contextual word problem/math story and to realize that the mathematics for this math story can be described with words such as the words they used during the introduction to the Math Talk/Number Talk and Mini-Lesson. Organize students into pairs or groups of 3 and give out Card Set A *Math Stories/Contextual Word Problems* and Card Set B *Explanations in Words*.

Students will cut the cards only when the pair/partnership has agreed to a match.

"Your task is to match the math story/contextual word problem with the corresponding explanation in words. Take turns with your partner to match that story with a word card. When you and your partner are in agreement with each other, then, and only then, may you cut out the two cards that match. If you cannot find a matching card, write your own using one of the blank cards.

"When you are finished, please find another pair of students to discuss your findings. If you disagree, you must all come to consensus or reach agreement. This will encourage and allow you to ask questions of each other and to think through your ideas. All the matches need to make mathematical sense to all of you."

Throughout this collaborative activity, students will be asked to occasionally create different representations. These experiences provide opportunities for you to assess students' understanding. In creating the representations, students will deepen their own understanding of the relationship between and among the different representations.

Please Note:

It is difficult for students to keep track of their cards. Thus, it is important that they have the opportunity to glue them onto an appropriately-sized piece of construction paper. If they do change their minds regarding a match[es] then they may cut the cards out with this construction paper backing and re-arrange before proceeding to match with Card Set C.

Another idea would be to laminate the cards and use blue painter's tape. This would provide you a set from year to year and also an easy way for students to re-arrange their cards without excessive paper and gluing issues.

The Lesson Continues Here for Both Beginnings

Collaborative Task: Matching ONE Math Story with ONE Explanation in Words

[15-20 minutes]

Materials needed: Card Set C.

Give each pair of students a copy of Card Set C *Number Sentences/Equations* and a glue stick. Ask students to organize and then glue their card matches for Card Sets A and B to a piece of large construction paper, poster paper or butcher paper. The number sentences/equations will help students translate from the math story/contextual word problem and words to the symbolic representation of number sentences/equations. This matching will help students solidify for themselves their understanding of the inverse relationship between multiplication and division. They will be able to see that in a division problem there is a missing factor and in a multiplication problem there is a missing product.

"Your task is to match the Math Stories from Card Set A and the Word Explanations from Card Set B with the Number Sentences/Equations from Card Set C. Take turns with your partner to choose a number sentence/equation from Card Set C to match one set of Cards A and B. Place the chosen Card C alongside the A and B card set on the table and explain to your partner how you know they match. If you cannot find a matching card, write your own using one of the blank cards.

"When you are finished, please find another pair of students to discuss your findings. If you disagree, you must all come to consensus or reach agreement. This will encourage and allow you to ask questions of each other and to think through your ideas. All the matches need to make mathematical sense to all of you."

Collaborative Task: Matching ONE Number Line Model with a matched Math Story, Explanation in Words and Number Sentence.

[15-20 minutes]

Materials needed: Card Set D.

The number line/measurement model helps students connect the quantities and operations of multiplication and division to real-world situations. With measurement or number line models, length is compared instead of area. Having manipulatives available provides more opportunity for trial and error, exploration and discussion.

Give each group of students the Card Set D *Number Lines/Measurement Models*.

“Your task is to match the Math Stories from Card Set A, the Word Explanations from Card Set B, and the Number Sentences/Equations from Card Set C with the Number Lines/Measurement Models from Card Set D.

“Take turns with your partner to choose a number line model from Card Set D to match one set of Cards A, B, and C. Place the chosen Card D alongside the A, B, and C card set on the table and explain to your partner how you know they match. If you cannot find a matching card, write your own using one of the blank cards.”

These posters will be displayed for the summary class discussion. As students do the matching and pasting or gluing, walk around and encourage students to explain why particular cards go together. Listen to their explanations and justifications and mark down any that you would like to highlight or draw attention to in the summarizing whole group discussion. Ask them questions and prompts similar to the ones suggested for the pre-assessment task, *Baking Cookies*.

PLENARY DISCUSSION [15-20 minutes]

First, give and collect the following prompt questions to the class:

1. **“Which representation made the most sense to you? Why?”**
2. **“Which representation was the most difficult for you to understand? Why?”**

Second, hold a whole class interactive discussion to review what has been learned over this lesson. Ask each pair of students to justify one of their groupings and explain to the whole class why the different types of representations convey the same quantity.

Third, after the completion of the presentations and collection of the individual responses, revisit with your students the Standards for Mathematical Practice listed in the beginning of this unit. Ask students to reflect upon these practices and write about how this lesson helped them practice one or more of these standards.

Individual Work [15-20 minutes]

Finally, give students a new copy of the task, *Baking Cookies*. Ask students to have another go at it. When students have completed their second attempt, pass out their original work to them and ask them to write a reflective paragraph about what they learned from the lesson which helped them to do better on their second attempt. Additionally they should also write about what they are still struggling to understand well. Make sure they write about their current understanding of the relationship between multiplication and division and the different representations of multiplication and division.

SOLUTIONS: The following table is for your convenience only.


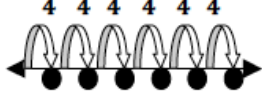
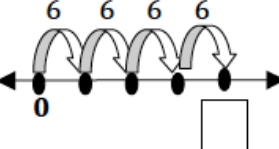
CONTEXTUAL	WORDS	EQUATION	NUMBER LINE
A1	B7	C1	D7
A2	B9	C2	D9
A3	B6	C3	D6
A4	B12	C4	D12
A10	B1, B14	C5	D1
A6	B10, B15	C6	D10
A7	B11	C7	D11
A8	B8	C8	D8
A11	B5	C9	D5, D6
A9	B4	C10	D4
A5	B2	C11	D2
A13	B3	C12	D13 and/or D14 Have a discussion
A12	B13	C13	D3

Rubric for *Baking Cookies*

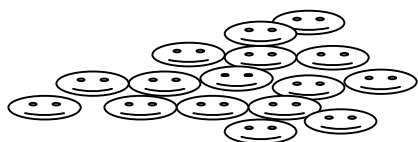
Baking Cookies	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • work with quantities in a contextual situation • use different representations for multiplication <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	
<p>1. Gives correct answer: 48 cookies</p> <p><i>Shows work such as: (award one point for each correct solution strategy)</i></p> <p>$4 \times 12 = 48$</p> <p>$12 + 12 + 12 + 12 = 48$</p> <p><i>(Accept any correct solution strategy including concrete, number line, and/or bar/area models.)</i></p>	<p>1</p> <p>2x1</p>
<p>2. Gives correct answer: 6 rows</p> <p><i>Shows work such as: (award one point for each of two different strategies)</i></p> <p>$30 \div 5 = 6$</p> <p>$5 \times 6 = 30$</p> <p><i>(Accept any correct solution strategy/representation including number line, array, or area model.)</i></p>	<p>1</p> <p>2x1</p>
<p>3. See <i>Answer Key</i> on next page showing multiple representations/models for this problem.</p> <p>4 correct answers</p> <p>3 correct answers</p> <p>2 correct answers</p> <p>1 correct answer</p>	<p>4 pts</p> <p>3 pts</p> <p>2 pt</p> <p>1 pt</p>
Total Points	10

Answers for Baking Cookies Part 3

Answers may vary.

	Problem 1	Problem 2	Problem 3
Equation/ Number Sentence	$\square \times 6 = 24$	$4 \times \square = 24$	$4 \times 6 = \square$
Contextual Word Problem/ Math Story	Jill baked 24 cookies for her friends. She had 6 cookies in each row in her pan, How many rows of cookies were in the pan?	Jill baked 24 cookies for her friends. She had 4 rows of cookies in each pan. How many cookies were in each row?	Jill baked cookies for her friends. She had 4 rows of 6 cookies in the pan How many cookies did she bake?
Explanation in Words	\square equal groups of 6 in 24	4 equal groups of \square in 24	4 times as big as 6 or 4 groups of 6
Number Line/ Measurement Model			

Performance Task
Baking Cookies



1. Jill is baking 'Happy Face Cookies' for herself and 3 special friends. She wants everyone to have 12 cookies each. How many cookies will she need to bake?

_____cookies

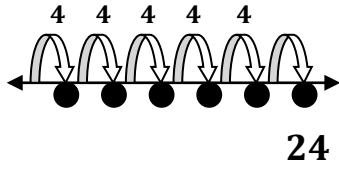
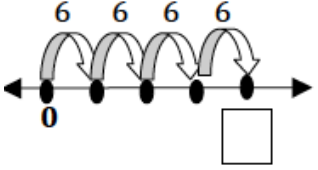
Show how you got your answer using two different ways.

2. Jill decides to give herself and each of her 29 classmates a cookie. When she bakes her cookies, she can fit 5 cookies in a row on her baking pan. How many rows of cookies will she need to make to bake all the cookies?

_____rows

Show how you got your answer in two different ways.

3. Jill decides to only bake cookies for herself and **3** special friends. She is using a pan that holds **4 rows of 6 cookies** or **24 total cookies in all**. There are **some pieces** of information missing in the chart below. You are to fill in the missing piece of information.

	Problem 1	Problem 2	Problem 3
Equation/ Number Sentence	$\square \times 6 = 24$	<i>(Write an equation/ number sentence)</i>	$4 \times 6 = \square$
Math Story/ Contextual Word Problem	<i>(Write a word problem)</i>	Jill baked 24 cookies. She had 4 rows of cookies in each pan. How many cookies were in each row?	Jill baked cookies. She had 4 rows of 6 cookies in the pan. How many cookies did she bake in all?
Explanation in Words	\square equal groups of 6 in 24	<i>(Write an explanation in words)</i>	4 groups of 6 equals
Number Line/ Measurement	<i>(Show a number line)</i>		

Card Set A– Math Stories/Contextual Word Problems

A1 Joe has 20 crayons. How many crayons does he put in each box if he gets 4 boxes for 20 crayons?	A2 Susie wants to give her 7 friends 8 pieces of candy each. How many pieces of candy will she need to buy?
A3 Polly’s mom has planted 15 plants in 5 rows. How many plants are there in each row?	A4 Sam’s dad bought 24 hotdogs for Sam and his 3 friends. How many hot dogs can they each have?
A5 Debbie wants to give her 5 friends 4 balloons each. How many balloons must she buy?	A6 Matt wants to give his 8 friends 9 baseball cards each. How many baseball cards must he buy?
A7 Sarah buys 6 pieces of bubble gum for 24 cents. How much does each piece cost?	A8 Caitlin buys 5 pies for \$20 each. How much does one pie cost?
A9 Daniel buys 3 dozen donuts. How many donuts does he buy?	A10 David wants to give his 4 friends 5 books each. How many books will he need?

A11 Sally baked 15 cookies. She put them in bags of 3 cookies. How many bags of cookies did she make?	A12 Mardi plants 36 plants in 3 rows. How many plants in each row?
A13	A14
A15	A16
A17	A18

Card Set B – Explanations in Words

B1 4 times as big as 5	B2 5 groups of 4
B3 10 groups of 3 and 2 groups of 3	B4 3 groups of 12
B5 <input type="text"/> number of equal groups of 3 in 15	B6 5 equal groups of <input type="text"/> in 15
B7 <input type="text"/> number of equal groups of 4 in 20	B8 20 divided into 5 equal groups
B9 7 groups of 8	B10 8 times as big as 9
B11 6 groups of <input type="text"/> in 24	B12 24 is divided into 4 equal groups of <input type="text"/>

B13 36 is divided into 3 equal groups of <input data-bbox="704 302 777 373" type="text" value="?"/>	B14 4 groups of 5
B15 8 groups of 9	B16
B17	B18

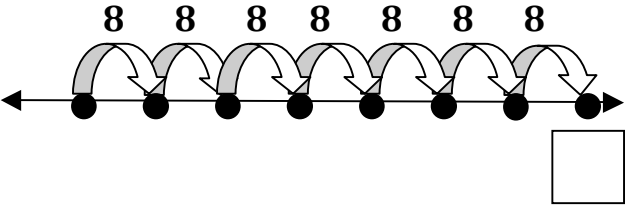
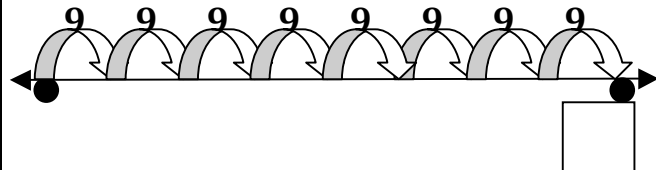
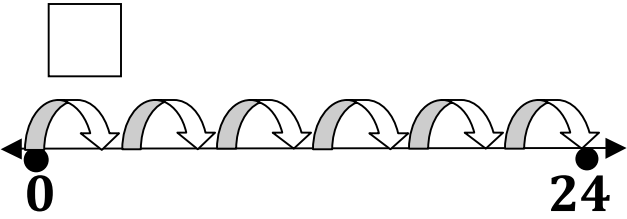
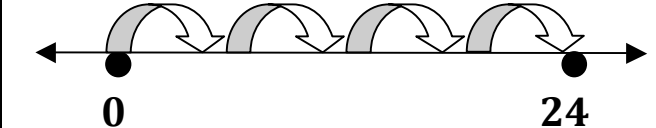
Card Set C-Equations /Number Sentences

C1 $20 \div \square = 4$	C2 $7 \times 8 = \square$
C3 $5 \times \square = 15$	C4 $24 \div 4 = \square$
C5 $4 \times 5 = \square$	C6 $8 \times 9 = \square$
C7 $6 \times \square = 24$	C8 $20 \div 5 = \square$
C9 $15 \div 3 = \square$	C10 $3 \times 12 = \square$
C11 $5 \times 4 = \square$	C12 $(10 \times 3) + \square = 36$

C13 $36 \div \square = 3$	C14
C15	C16
C17	C18

Card Set D - Number Line Model

<p>D1</p> <p>5 5 5 5</p> <p>0 <input type="text"/></p>	<p>D2</p> <p>4 4 4 4 4</p> <p>0 <input type="text"/></p>
<p>D3</p> <p><input type="text"/></p> <p>0 <input type="text"/> 36</p>	<p>D4</p> <p>12</p> <p>0 <input type="text"/></p>
<p>D5</p> <p>3 <input type="text"/> ? groups of 3</p> <p>0 <input type="text"/> 15</p>	<p>D6</p> <p><input type="text"/></p> <p>0 <input type="text"/> 15</p>
<p>D7</p> <p>5 <input type="text"/> ? groups of 5</p> <p>0 <input type="text"/> 20</p>	<p>D8</p> <p><input type="text"/></p> <p>0 <input type="text"/> 20</p>

<p>D9</p> 	<p>D10</p> 
<p>D11</p> <p><input type="text"/></p> 	<p>D12</p> <p><input type="text"/></p> 
<p>D13</p>	<p>D14</p>